PHYSICS FOR MEDICAL STUDENTS

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E. & S. LIVINGSTONE
17 TEVIOT PLACE
1928

13976
194

530.0246
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Made and Printed in Great Britain.

PREFACE

THE scope of this book is intended to cover all that is generally expected of a student taking the pre-medical examination in Physics in this country. It is inevitable with the growth of both subjects, medicine and physics, that certain aspects of physics are seen to become more and more valuable in their application to medicine, and although it is impossible for educational authorities to change the scope of the subject with every advance in physical science, yet there is a gradual change occurring which is reflected in examination papers, showing that more attention is directed to those branches of physics which have a direct bearing on medical science.

One aim of the book has therefore been to cut down to a minimum subjects such as Sound and Magnetism and to go into greater detail in the other parts of the subject, giving due weight to Mechanics, without some knowledge of which the subject of physics cannot be properly studied.

Another aim has been to indicate throughout the text some direct applications of physics in medicine, hoping in this way to create an interest among medical students in a subject which is often rather disliked.

A systematic course of physics includes lectures, practical work, and the working through of numerical examples. An excellent selection of these examples is given in the many editions of Jones's "Examples in Physics," and no attempt is made to include them to any appreciable degree in the text here. As far as practical work is concerned, this is best done unc personal supervision in the laboratory, so that detail descriptions of experiments are omitted and attenti focused upon the general principles of the subject.

I have much pleasure in thanking Mr B. D. Watters, M.Sc., and Miss G. M. Scott for reading t proofs; Mr Watters again for the original photograp in Figs. 100 to 102, showing the distribution of magnet fields in special cases; Miss Clephan for all the care s has given to the drawings; Mr Cuthbert Andrews f the illustration of an X-ray tube; and Messrs Chapma & Hall for permission to reproduce several illustratio from the book "Physics in Medical Radiology."

THE AUTHOR.

London, October 1928.

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ERRATA

Page 23, line 11. *For* w *read* W.
line 14. *For* 26 *read* 32·5.

Figs. 71 and 73. Dispersion should also be shown *within* the prisms.

Physics for Medical Students

CHAPTER I

MECHANICS

The subject of Physics has largely to do with measurement, and differs in this way from the more descriptive sciences. This is not to say that the description of phenomena is outside the scope of physics; far from it, because one of the chief aims is to understand how things happen; how, for instance, some bodies float in water, how sound makes its way from a sounding body to our ears, how electricity travels through a circuit, and so on. The difficulty of explaining these things becomes more and more manifest the more the phenomena themselves are probed, and in the course of this probing it is found that more and more comes to light if we can, as it were, measure the effects as we go on, so that measurement plays an enormous part in the study of physics and in its advancement.

The fundamental units of measurement are those of mass, length, and time. These conceptions are very familiar to us, although we may never have learnt to use them very exactly. It was agreed in 1873 by physicists to use certain units for measuring these quantities, and they are specified as follows :—

The unit of mass is called the gram (or gramme) and is one thousandth part of a mass of platinum kept in Paris and called the Kilogram des Archives.

The unit of length is called the centimetre and is one hundredth of the distance between the ends of a certain bar of platinum at 0° C., also kept in Paris.

The unit of time is the second and is one eighty-six-thousandth part of the time taken for the earth to rotate upon its axis.

In physics we are very quickly called upon to measure and compare forces, and our conception of force is based on the early work of Newton upon the motion of material bodies.

The simplest way of studying motion is to think of the motion of a particle, and by a particle we mean a mass so small that it can be considered as concentrated at a point. In this way we get over the difficulty, which has, of course, to be met later on, of thinking about the different motions which different parts of the same body may have at the same moment; for instance, a wheel rolling along the ground has a complicated motion.

Let us consider a particle which is moving in a straight line and going from A to B. If it travels with uniform velocity v for a time t, the distance s travelled is obviously $s=vt$. If, however, it is not travelling with uniform velocity, it is said to be accelerated we reckon this positive if it is gaining, but negative if it is losing velocity. The only case of acceleration which will be considered here is when it is uniform, *i.e.*, when equal amounts of velocity are gained or lost per unit o time. Let this acceleration be a, and let the initia velocity of the particle be μ. Then a units of velocity ar added on every unit of time.

The velocity at the end of the 1st second will be $\mu+a$.
,, ,, ,, 2nd ,, ,, $\mu+2a$.
,, ,, ,, 3rd ,, ,, $\mu+3a$.
,, ,, ,, t seconds will be $\mu+at$.
Hence the final velocity is given by $V=\mu+at$ . . (1

The next question is, how far will it have gone i this time t? This is most easily seen by the help o Fig. 1. If we mark off on the vertical axis, the velocit at any time, beginning with the point E corresponding to an initial velocity μ, we find that the velocities at the end of 1, 2, 3 . . . t seconds lie on the straight line ED,

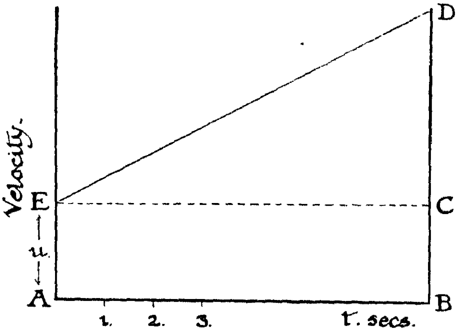

FIG. 1.

and since the space traversed by the particle is always the velocity (if constant) multiplied by the time, we can see that the area of the figure AEDB will give us a measure of the distance travelled.

The area of Fig. AEDB is AECB + EDC

$= ut + \frac{1}{2}\text{EC.DC}$

$= ut + \frac{1}{2}(t(v-u))$, v being the final velocity at D

$= ut + (\frac{1}{2}vt - \frac{1}{2}ut)$

$= \frac{1}{2}(u+v)t.$

But we have seen from equation (1) that $v = u + at$; so substituting this value for v, we have for the distance

$$\mathrm{S} = \tfrac{1}{2}(u+u+at)t = ut + \tfrac{1}{2}at^2 \quad . \quad . \quad . \quad (2)$$

The final velocity of the particle can be found in terms of u, a, and S by eliminating the time in the following way :—

$$v = u + at \quad \therefore \; t = \frac{v-u}{a}$$

$$\therefore \; \mathrm{S} = u\frac{(v-u)}{a} + \tfrac{1}{2}a \; \frac{(v^2 - 2uv + u^2)}{a^2}$$

$$\therefore \; 2a\mathrm{S} = 2uv - 2u^2 + v^2 - 2uv + u^2 = v^2 - u^2,$$

or

$$v^2 = u^2 + 2a\mathrm{S} \quad . \quad . \quad . \quad . \quad (3)$$

With the help of these three equations we are able to solve any case of uniformly accelerated motion of a particle.

Example.—A particle starts with a velocity of 10 ft. per sec. and has an acceleration of 20 ft. per sec. per sec. How far will it go in a minute, what velocity will it then have, and what is its average velocity for the whole distance traversed ?

Using $S = ut + \frac{1}{2}at^2$ and putting in values, we have

$$\text{Distance} = 10 \times 60 + \tfrac{1}{2} . 20 \times 60^2 \text{ ft.} = 6 \cdot 93 \text{ miles.}$$

The final velocity is got from the equation

$$v = u + at = 10 + 20 \times 60 = 1210 \text{ ft. per sec.}$$

The average velocity is given by $\frac{u+v}{2}$; hence in this case it is $\frac{10+1210}{2} = 610$ ft. per sec.

In specifying an acceleration, time has to be stated twice; this is because it is a rate of change of velocity, and a velocity requires time in its specification.

We know that things dropped usually fall to the ground, and most bodies do so very quickly. Up till the time of Galileo it was thought that the heavier the body, the quicker it would fall. Galileo (1564-1642) dropped weights of different substances from the leaning tower of Pisa and showed that they all reached the bottom in the same time, or very nearly so; the small differences he rightly attributed to the different degree of air resistance they encountered.

The cause of this fall to the earth is the earth's attraction, and the particular force acting is called the gravitational force. We shall see in a later section that the effect of force applied to a body is always to produce or to tend to produce an acceleration of the body, and in this case the acceleration of a body let fall is called the gravitational acceleration. It is a very big acceleration, and in London has a value of about 32 ft. per sec. per sec., or 981 cm. per sec. per sec. Its value varies slightly over the surface of the earth, being greatest at the poles and least at the equator. It is generally referred to as g.

Example.—How far will a body go in 10 seconds if let fall under gravity ? In this case the initial velocity $u=o$.

Using equation

$$S=ut+\frac{1}{2}at^2=o+\frac{1}{2}32\,.\,10^2=1600\text{ ft.}$$

It is very inconvenient to measure g by any observations of this kind. It is much more simply and accurately done by means of the device known as a simple pendulum. This consists of a small leaden ball suspended by means of a thread which is fixed to a rigid support. If the bob of the pendulum is pulled aside through a small angle and let go, it swings to and fro and is said to execute simple harmonic motion. Even though the resistance of the air damps down the swings, the time for a complete to and fro motion is not changed. The time of swing T depends only on the length of the pendulum, *i.e.*, the distance from the point of support to the centre of the bob and on the gravitational acceleration.

The expression connecting these quantities is :—

$$T=2\pi\sqrt{\frac{l}{g}}.$$

Experimentally the time for 50 or 100 complete swings is found and the time T for one swing deduced from the result ; hence g is found.

We cannot actually measure a velocity by simply finding the distance passed over in a given time ; by this we only get the speed, which has no reference to direction, and this is called a scalar quantity. To determine the velocity the direction is also required, and for this reason a velocity is called a vector quantity. The length of a line is often taken to represent the speed of a body ; the direction that the line is drawn with reference to some axes gives the additional data required to specify the velocity. Now any change in velocity means an acceleration, so that accelerations are also vector quantities. If a body changes its direction without altering its speed, its velocity nevertheless changes, and therefore it is accelerated. A good example of this is to take a simple pendulum and make the bob move round in a horizontal circle; its speed may be quite uniform, but its velocity is changing because its direction is being altered at every instant, therefore it is being accelerated. In what follows we shall see that this implies that some force is acting all the time on the bob of the pendulum.

To Newton (1642-1727) we owe the statement of three laws of motion which are true for all the motions considered here. These laws are due more to Newton's extraordinary insight into the properties of matter than to any elaborate set of measurements; in fact it is questionable whether Newton ever actually devised any experiment to test the first of these laws.

1st Law.—That every body perseveres in its state of remaining at rest or of moving uniformly in a right line, except in so far as it is compelled by impressed forces to change its state.

2nd Law.—That change of motion is proportional to the impressed moving force and takes place along the right line in which that force is impressed.

3rd Law.—That reaction is always opposite and equal to action, or that the actions of two bodies mutually on one another are always equal and tend in opposite directions.

The first law is often called the law of inertia. Things stay exactly as they are, at rest or in uniform motion, unless some force acts on them. A force applied to a body at rest does not necessarily set it in motion; but it tends to do so, and our conception of force is essentially this, that when applied to a body it tends to accelerate it.

The second law gives us a means of measuring and comparing forces. Let us suppose that a force F applied to a mass m gives it an acceleration; then a force twice as big will give the same mass twice the acceleration. There is, in fact, a strict proportionality between the force applied to a body of mass m and the acceleration a that it produces. If the same force acts on a smaller mass, it is found to accelerate it more; if on a large mass, then the acceleration is proportionately less.

This is expressed by saying that force is proportional to the product of mass and acceleration, or $F = ma$, the constant of proportionality being made unity.

Unit force is that force which, acting on unit mass, gives it unit acceleration. On the C.G.S. (centimetre, gram, second) system of units this force is called the dyne and is that force which, applied to a mass of 1 gram, produces upon it an acceleration of 1 cm. per sec. per sec.

On the F.P.S. (foot, pound, second) system of units, this force is called the poundal, and is that force which, applied to a mass of 1 lb., produces upon it an acceleration of 1 ft. per sec. per sec. These are called the dynamical units of force. Engineers make more use of gravitational units of force, and these are derived in the following way.

Newton showed that bodies are attracted to the earth by virtue of what he called the gravitational force. This kind of force is not peculiar to our earth, but is supposed to be shared by matter wherever it may be. Newton showed that this force keeps the moon circling round the earth and all the planets round the sun.

The force with which the earth pulls on a body is called the weight of a body, and we have seen that a body let go near the surface of the earth has an acceleration g; if the mass of the body is m, then the force on it is given by $F = mg$; hence if W is the weight, we have $W = mg$.

Now, if we consider a mass of 1 lb. falling freely under gravity, it is accelerated by about 32 ft. per sec. per sec., and so this force which we call the weight of a pound is about thirty-two times as big as the poundal. This is the gravitational unit of force, namely, the weight of a pound, and is often more convenient to use than the much smaller force, the poundal. The gravitational unit on the C.G.S. system of units is the weight of 1 grm.; this is equal to about 981 dynes.

It is important to distinguish carefully between mass and weight. The mass of a body is invariable and does not change with its position on the earth; but the weight of a body does, and if we could take a mass sufficiently far away from the earth, it would cease to have any weight.

When we use an ordinary chemical balance to weigh things, we are really only comparing the masses in the two scale pans. Equal masses "weighed" in this way at London and at the Equator would appear the same, though we know their weights would really differ slightly.

A spring balance, however, does weigh bodies, though generally rather inaccurately. In this case we are balancing the real weight of the body by the pull of a contracting spring.

A body in motion has momentum, and this is defined as the product of its mass into its velocity or mv.

The C.G.S. unit of momentum is that of a gram travelling with a velocity of 1 cm. per sec. No special name is given to this unit. We can, of course, only change the momentum of a body if we change its velocity (mass being unalterable), and we can only do this by exerting a force on it; so if we think of a whole lot of masses travelling about in different directions with constant velocity, then nothing can change the total momentum of this system unless an external force is brought to bear on it. Suppose, however, that no such thing is done and the bodies are considered immune from any outside forces. Newton's third law implies that the momentum of this collection of moving bodies must for ever remain unaltered. Such a conclusion is sometimes expressed by the term "the conservation of momentum." Let us suppose that two of the bodies in this moving collection happen to have a head-on collision; certainly their velocities will alter, but the principle states that their total momentum after the collision is exactly the same as it was before. If m_1, m_2 are the masses, and v_1, v_2 the velocities before and $v_1{}^1$, $v_2{}^1$ the velocities after collision, we may write:—

$$m_1v_1 + m_2v_2 = m_1v_1{}^1 + m_2v_2{}^1.$$

A good example of the use of this principle is in finding how a gun recoils when a shot is fired from it. Initially we consider the gun and shot at rest, therefore the system gun + shot has no momentum. Hence when the shot is fired, the system gun + shot can still have no momentum; the shot has got a velocity and the gun will have a velocity, the velocity of recoil, but they are in opposite directions, and the momentum of the gun just cancels the momentum of the shot.

Example.—A 10-ton gun at rest fires a shot of 224 lbs. with a velocity of 1,000 ft. per sec.; find the velocity with which the gun recoils.

Momentum of gun and shot before firing
= momentum of gun and shot after firing.

$$0 = 10 \times 2240 \times \text{velocity of recoil} - 224 \times 1000.$$

$$\therefore \text{ Velocity of recoil} = \frac{224 \times 1000}{10 \times 2240} = 10 \text{ ft. per sec.}$$

When a force moves its point of application it is said to do work, and the amount of work done is measured by the product of the force into the distance moved in the direction of the force.

Thus $\text{Work done} = \text{force} \times \text{distance},$
or $W = F \,.\, S.$

The unit of work in the C.G.S. system is called the erg, and is the work done by unit force moving through unit distance, *i.e.*, a dyne through a centimetre. The unit of work on the F.P.S. system is called the foot-poundal, and is the work done by unit force moving through unit distance, *i.e.*, a poundal through a foot. Engineers find this rather a small unit of work and generally use a gravitational unit of work, namely, the foot-pound, which is the work done by the weight of a pound moving through 1 ft.

Our impressions of work are very often influenced by the rate at which the work is done. If we take three flights of steps at a run, we do no more work than if we take them leisurely, though our impression is otherwise. In the former case, however, we have to exert much more power. Power is defined as the rate of doing work.

The engineer Watt made an estimate of the rate of work which could be expected from an "average" horse and arrived at a figure which has been adopted as a horse-power (H.P.).

$$\begin{aligned}1\text{ H.P.} &= 550\text{ ft.-lbs. per sec.}\\ &= 33{,}000\text{ ft.-lbs. per min.}\end{aligned}$$

The term energy is used to denote the capacity for doing work. If a system has work expended on it, then we say that the system has gained energy. If, for instance, we raise a weight to some height above the surface of the earth, we do work against the attractive force of the earth and give the weight energy; it retains this energy until such time as we use it. By virtue of its position, with respect to the earth, we say it has potential energy and the amount it possesses is in exact proportion to the amount of work done in getting it to its position. If the weight be W and the height to which it is raised is h, then :—

$$\begin{aligned}\text{Potential energy} &= \mathrm{W}h \text{ gravitational units}\\ &= \mathrm{M}g\,.\,h \text{ dynamical units of work.}\end{aligned}$$

The units of energy are those of work, so if we measure the work in C.G.S. units, the energy will be in these units, viz., ergs; if in F.P.S. units, the energy will be in foot-poundals or foot-pounds. In many cases when work is done upon a body it is set into motion and the energy it now possesses is called kinetic.

Let us suppose a body of mass M is travelling with a velocity V, and we want to find how much kinetic energy it has. If it has energy or capacity for doing work, then this will be just equal to the work we should have to do in order to stop it. Let the body be stopped by applying a force F in the opposite direction to that in which the body is travelling, as in Fig. 2.

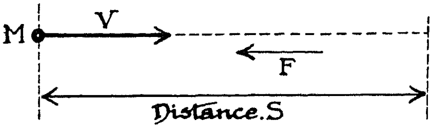

Fig. 2.

The immediate action of the force F on the mass is to give it a backward acceleration $a=\frac{F}{M}$. This will, of course, gradually slow the body down. Let S be the distance it goes before coming to rest, and therefore having no kinetic energy.

From the relation $V^2=u^2+aS$

we have $$0=V^2-2aS=V^2-2\frac{F}{M}.S.$$

$$\therefore \tfrac{1}{2}MV^2=FS$$
$$=\text{Work done by the applied force}$$
$$=\text{Kinetic energy of the body.}$$

It is a matter of experience that we cannot get more work out of a machine than we put into it; actually we never get quite so much because of frictional and other losses. This general experience, combined with accurate observation of the way in which energy can be converted from one form into another, has led to the recognition of a very important principle known as "the conservation of energy," which states that energy cannot be created or destroyed, but that it can be changed so that it alternately appears in potential or kinetic form.

To return to the case of a weight lifted off the ground: the higher it is raised, the greater its potential energy becomes. Now, suppose that the weight is let go, it drops towards the earth with velocity increasing as it goes, its kinetic energy increasing as its potential energy diminishes. Just before contact with the earth its energy is practically all of the former and none of the latter kind; a moment later it strikes the earth and comes to rest, its kinetic energy apparently gone also. This looks like a direct contradiction of energy being conserved, because it appears to have completely vanished. Accurate observation has, however, shown that heat is produced on impact of the body with the earth, and we shall see later that there are good reasons for believing that heat is a kind of motion, although this is generally quite invisible. So that the visible kinetic energy has really been converted into the invisible motion that we call heat; a little of the energy will very likely be converted into sound energy, and this is also invisibly kinetic. Once the stage in scientific thought had been reached that warranted a belief in the conservation of energy, there was little excuse for the genius of inventors being spent in trying to devise machines which would give perpetual motion. This inventiveness is now directed towards getting nearly as much energy out of a machine as is put into it, not more.

We have seen that, in order to specify a velocity correctly, we need to state the direction in which the body is travelling; if we reckon this positive when the body is going, say, from left to right, then it will be negative when from right to left, and similarly with accelerations and forces which are also vector quantities. The kinetic energy of a body is $\frac{1}{2}MV^2$, so whether the velocity is positive or negative we see that the kinetic energy, because of the squared term, is always positive; we cannot have a body possessing a negative amount of energy. Let us now consider a body travelling with velocity V in a north-easterly direction: let us mark off along the line OA in Fig. 3 a length proportional to the speed of the body, the angle θ determining its direction. Now, although the body is travelling neither due north nor due east, it has a tendency in both of these directions; for, at the end of unit time, the body having arrived at A, it will be both more northerly and more easterly than it was at O. We may resolve the velocity V into two components, one of which would be given by ON, the other by OE. Conversely, if the body had simultaneously the velocities represented by ON and OE, then it would have a resultant given by OA.

Many problems are easily solved by this method of resolving a velocity, acceleration, or force into components and vice versa by getting the resultant of two or more velocities, accelerations, or forces acting simultaneously on a body.

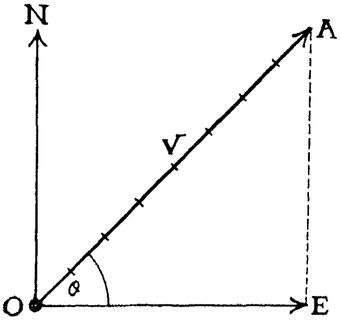

FIG. 3.

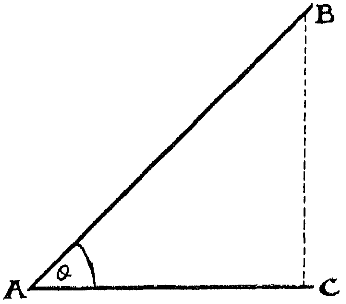

FIG. 4.

As we shall frequently have occasion to refer to certain trigonometrical relations, these will be mentioned here. If the two lines AB and AC in Fig. 4 include any angle θ and from any point B, a perpendicular is drawn to AC, then :—

$$\frac{BC}{AB}=\frac{\text{perpendicular}}{\text{hypotenuse}}=\text{sine }\theta\text{ or }\sin\theta.$$

$$\frac{AC}{AB}=\frac{\text{base}}{\text{hypotenuse}}=\text{cosine }\theta\text{ or }\cos\theta.$$

$$\frac{BC}{AC}=\frac{\text{perpendicular}}{\text{base}}=\text{tangent }\theta\text{ or }\tan\theta.$$

If we take an equilateral triangle ABC in Fig. 5 and drop a perpendicular BD, the angles of the △

BCD are 30°, 60°, and 90°. Let DC = 1 unit of length.

Hence $\sin 0° = 0$ and $\cos 0° = 1$.

$\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$.

$\sin 60° = \frac{\sqrt{3}}{2}$ and $\cos 60° = \frac{1}{2}$.

$\sin 90° = 1$ and $\cos 90° = 0$.

Values for any other angle can be got from mathematical tables.

The reader will have no difficulty in obtaining values for an angle of 45° by constructing an isosceles triangle and proceeding in the above way.

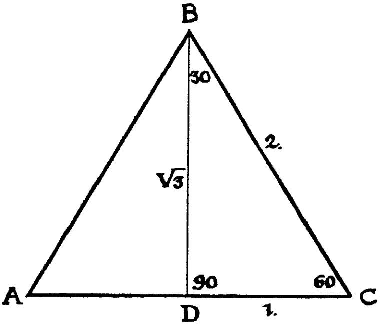

Fig. 5.

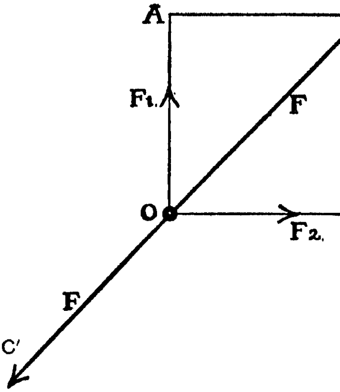

Fig. 6.

Turning now to Fig. 3 we see that the components of the velocity V, namely ON and OE, could equally well be written V sin θ and V cos θ. Let us now consider a particle which is acted on by forces F_1 and F_2 which can be represented in magnitude and direction by the lines OA and OB in Fig. 6. Their resultant will be OC, the diagonal of the parallelogram OACB, which is called the parallelogram of forces. The particle O will, if free to do so, move in the direction OC. If we wanted to prevent this, it would be sufficient to use another force F and make it act along OC′ exactly opposite to the direction OC. If we do this, the particle at O will not move in spite of three forces, F_1 and F_2 and F (along OC′), acting upon it; the necessary condition is that one of them balances the resultant of the other two.

It is necessary to discuss the equilibrium of bodies, and by equilibrium we mean that however many forces there may be acting on a body, the whole effect is that the resultant force is zero, so that the body acted on remains at rest or in steady motion, *i.e.*, no acceleration.

Say that a particle at O were subject to three velocities simultaneously and that the lengths OA, OB, and OC′ represent in magnitude and direction these three velocities; then the particle O will remain at rest. We see that the triangle OAC, which is called the triangle of velocities, is such that its sides taken in order represent the three velocities. A similar condition holds when we can find a triangle of forces to represent three forces acting simultaneously on a particle, *i.e.*, the particle remains in equilibrium.

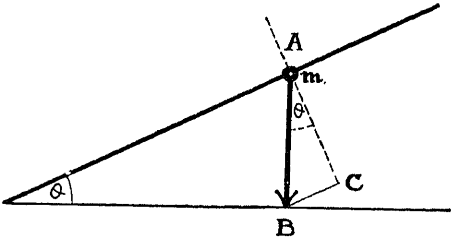

FIG. 7.

INCLINED PLANE.—Say that we have a mass on an inclined plane, as in Fig. 7. The effect of gravity is vertically downwards, and were it not for the plane the mass would, of course, have the acceleration g vertically downwards. We may resolve this acceleration into two components, one along the plane and the other vertically to it.

Let AB represent g, draw AC perpendicular to the plane, and drop BC perpendicular to AC.

Then $CB = AB \sin \theta$

$= g \sin \theta$, is the acceleration of the mass m down the plane.

However slightly the plane is tilted there is a positive acceleration down the plane, yet experience tells us that a body can stay where we put it on an inclined plane, even though the plane is tilted to quite a large angle; this is due to friction. Friction is a force which tends to stop the relative motion of bodies; practically speaking, whenever we try to move bodies about, frictional forces come into play and we have to do extra work in overcoming these forces. Friction is evident in the motion of solids, liquids, and gases.

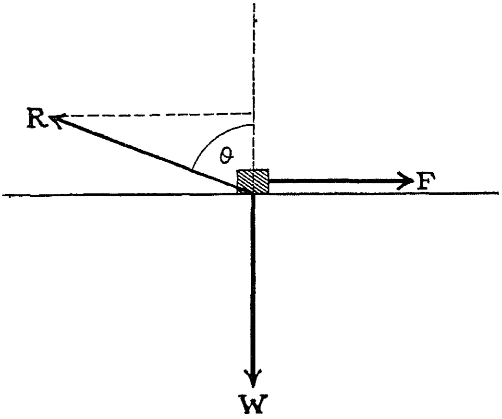

FIG. 8.

Say that we have a small block of wood on a wooden plank, as in Fig. 8. If we apply a horizontal force F to the block, then however small F may be, it will set the block into motion if the wood surfaces are perfectly smooth. This is never found to be the case and, if we begin with a force F, which is small, we may gradually increase it without causing the block to move; then suddenly it does move and we say that the limiting amount of friction between the two surfaces has been reached. Theoretically, we may speak of a perfectly smooth surface and we look upon it as one which can only give a reaction R which is perpendicular to its surface. In a rough surface this reaction R is not perpendicular. In the case in question we see that just before the body starts into motion under the action of the force F, the vertical component of R balances the weight W and the horizontal component of it balances F.

The ratio $\frac{F}{W}=\frac{R \sin \theta}{R \cos \theta}=\tan \theta=\mu$ the coefficient of friction.

We may take as examples of motion on smooth and rough surfaces, the case of a mass M on a horizontal table attached by a string to a scale pan hanging vertically and carrying a mass m, as in Fig. 9.

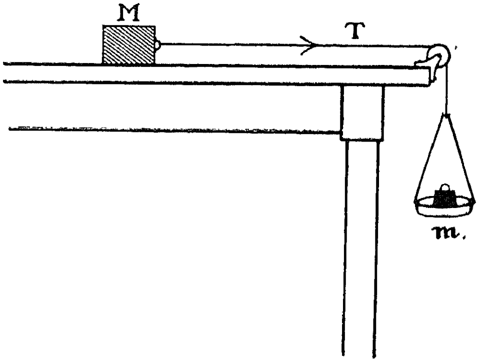

FIG. 9.

If the table is smooth, *i.e.*, no friction, the mass M moves to the right with an acceleration given by the equation F = total mass moved × acceleration.

$$\therefore \text{ Acceleration}=\frac{\text{Force}}{\text{Mass moved}}=\frac{mg}{M+m}.$$

No matter how small m is, M will move with some, though it may be a very small, acceleration.

If the table is rough and μ the coefficient of friction, then M will not begin to move until the tension T of the string caused by the pull due to the weight of the mass m is equal to μMg, because $\mu=\frac{T}{Mg}$ by definition. If m is increased beyond this value, the mass M is accelerated and so the effective force causing motion will be $mg - \mu Mg$.

$$\therefore \text{ Acceleration} = \frac{(m - \mu M)g}{m + M}.$$

If $\mu = \cdot 1$ (a very small degree of friction), M = 100 grm. and $m = 50$ grm., the acceleration would be

$$\frac{(50 - 10)g \text{ cm. per sec. per sec.}}{150} = 261 \text{ cm. per sec. per sec.}$$

Moment of a Force.—When a force is applied to a body, the effectiveness of the force depends very much on the way it is applied. A small force applied in the usual way is sufficient to open or close a door, but to do the same thing by applying a force near to the hinges means that we have to use a much bigger force.

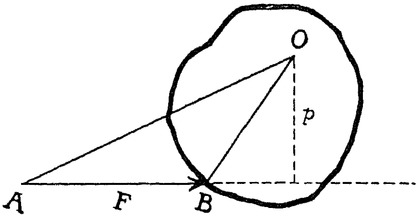

Fig. 10.

A body which preserves its shape and size whatever we do to it is called a rigid body. Let O be any point in a rigid body (Fig. 10) to which we apply a force F, then the moment of F with respect to the point O is the product of F and p where p is the perpendicular distance from O on to the line of action of F. If AB represent F in magnitude and direction, then the moment of F about O is Fp.

Joining OA and OB, we see that

$$\triangle OAB = \tfrac{1}{2}p \text{ . } AB.$$

Hence the moment of F about O is given by twice area of $\triangle$OAB.

A force directed through O has no moment at all about O.

We have seen that two forces inclined to one another at any angle may be said to have a resultant, the numerical value of which is given by the parallelogram of forces. It may easily be proved that the moment of the resultant about any point = sum of the moments of the individual forces about the same point. (*Vide* Porter's "Intermediate Mechanics," p. 150.)

We may use this important theorem to get the resultant of any number of parallel forces.

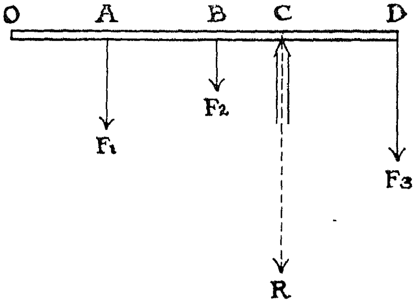

FIG. 11.

Suppose that three forces F_1, F_2, and F_3 are applied to a rigid bar, as in Fig. 11. Then if we take moments about any point O, the moment of the resultant is equal to the sum of the moments of the individual forces.

Let the resultant R act at C; this point is found as follows:—

$$R \times OC = F_1 \times AO + F_2 \times BO + F_3 \times DO,$$

but

$$R = F_1 + F_2 + F_3,$$

$$OC = \frac{F_1 \times AO + F_2 \times BO + F_3 \times DO}{F_1 + F_2 + F_3}.$$

If we want to prevent the bar moving vertically downwards, all we have to do is to apply a force equal to R vertically upwards at C. If a single support be placed at C, the rod will balance.

The most important case of this kind that we have to consider is that involved in considering the weight of a rigid body. Let AB (Fig. 12) represent a heavy uniform bar. We can think of this as being made up of small equal pieces, and the weight of each little piece acts vertically downwards.

The result is that we have a very large number of equal and similarly directed forces. They will have a resultant equal to their sum; this force is, of course, the weight of the bar, and the point in the bar at which the weight acts can be found by the rule just given. The point is called the centre of gravity, C.G., and in all uniform bodies which have geometrical symmetry, its position can be seen at a glance. It will in this case be at the mid-point of the bar. If the bar is made into a circular strip, the C.G. is at the centre of the circle (not in the body at all). If the circular strip is revolved about a diameter making a sphere, the C.G. is still at the centre, and so on. The C.G. of a rectangular plate will by symmetry again be at the intersection of the diagonals, and for a cube it will be at the centre of the cube.

The weight of a body, then, for all practical purposes acts through its C.G. A single force equal to its weight acting vertically upwards through the C.G. will, of course, balance the body.

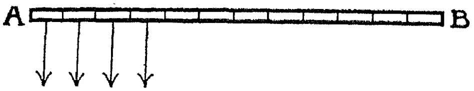

Fig. 12.

The equilibrium of a body is largely determined by the position of its C.G. A body is said to be in stable equilibrium if when it is given a slight displacement and the displacing force then removed, the body tends to come back to its old position.

If it tends to stay exactly in the displaced position, the equilibrium is neutral; whereas if it tends to go further and further from its displaced position, it is in unstable equilibrium. A cone resting on its base, its side, or its apex may serve as illustrating the three types of equilibrium.

Machines.—The principle of moments is of great use in the study of machines, the simplest of which is the lever. Let AC (Fig. 13) represent a rigid bar resting on a fulcrum B supporting a weight W at A; then a force F will support this weight if the moment of F about B is equal and in the opposite direction to the moment of W about B,

i.e., $$F \times BC = W \times AB, \text{ or } \frac{W}{F} = \frac{BC}{AB}.$$

The ratio $\frac{W}{F}$ which represents the ratio of the weight or resistance overcome to the force applied is called the mechanical advantage of the system.

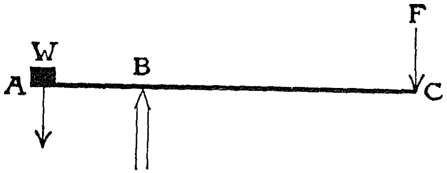

Fig. 13.

If $\frac{W}{F}$ is greater than 1, we have real mechanical advantage; if it is less than 1, the contrivance would be working at a mechanical disadvantage.

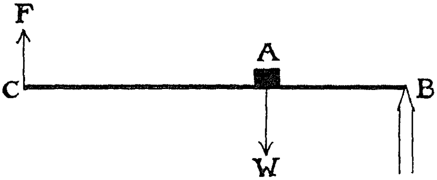

Fig. 14.

Say that as a variation we put the support, or fulcrum as it is called, at one end of the bar, as in Fig. 14, and the weight W between it and F, then for a balancing condition we have as before

$$F \times CB = W \,.\, AB, \text{ or } \frac{W}{F} = \frac{BC}{AB},$$

and this must always be greater than 1, and therefore there is always mechanical advantage.

Finally, let us put F between the fulcrum and the weight to be supported, as in Fig. 15. In order to balance the rod, we have

$$F \times CB = W \, . \, AB, \text{ or } \frac{W}{F} = \frac{BC}{AB}.$$

In this type of case the machine will always work at a mechanical disadvantage. In spite of this, machines are sometimes run at a mechanical disadvantage because of their convenience.

It is at first sight rather extraordinary that practically all the movements of the body in which there is leverage are carried out in such a way as to involve mechanical disadvantage. Two typical cases are illustrated in Figs. 16 and 17. In supporting a weight in the outstretched hand, the elbow-joint is the fulcrum and the balancing force is supplied by the tendons attached to muscles which, by their contraction, cause a pull in these tendons. The obvious mechanical disadvantage is accentuated by the fact that the tendons pull in the forearm at a very considerable angle to the vertical with the arm fully extended; if this angle is about 80°, the pull on the tendon has to be very large if W is appreciable, since it is only the vertical component of the force that can help to balance W.

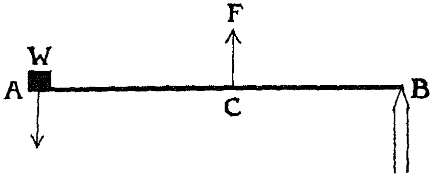

Fig. 15.

In the second case, which illustrates the act of standing on tip-toe, the arrangement is a little more complicated, for there is a tendon on both sides of the shin bones, the pulls on which have to be very considerable in order to overcome the moment of the whole weight of the body about the fulcrum. If the distance from the point of contact of the toe with the ground to the ankle-joint is taken as $6\frac{1}{2}$ in. and the distance from this joint to the attachment of the tendon of Achilles be taken as 2 in., we may calculate the pull on this tendon by taking moments about the fulcrum, *i.e.*, where the toe touches the ground.

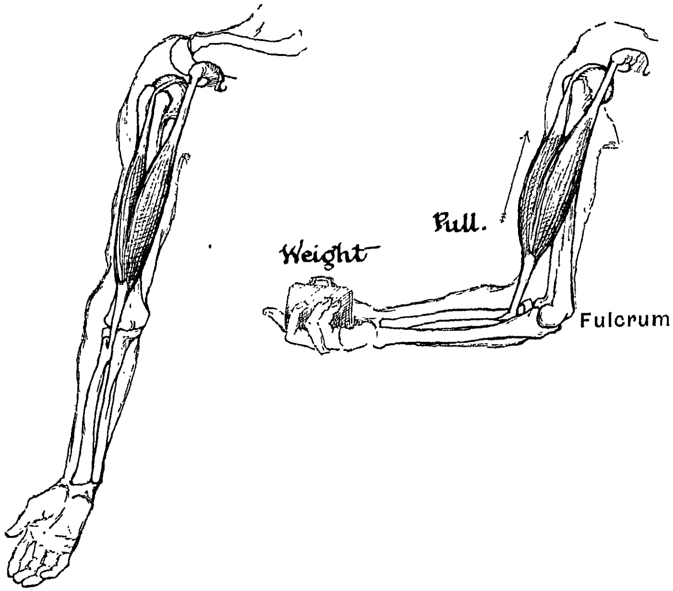

FIG. 16.

If W = force due to the body acting vertically downwards through the ankle-joint,
p = vertical pull on the tendon,
r = reaction from the ground,

we have $$W \times 6\tfrac{1}{2} = p \times 8\tfrac{1}{2},$$
also $$p + r = w.$$ W

If we consider this act by a man of 10 stone, we see that the pull on the tendon amounts to no less than 26 stone.

5 Returning to the simple lever shown in Fig. 13, the force F balances the weight W; with a long lever having the fulcrum very near the weight, we can

balance a big weight by applying a very small force. We may not argue from this that we could actually raise the weight through a distance by the expenditure of less work if we used such a machine than by direct lifting. Work is measured by the product of the force into the distance moved, and if the lever were put into working action we should find that, in order to raise W through a small distance *d*, we should have to move the force through a much bigger distance *d′*, so that

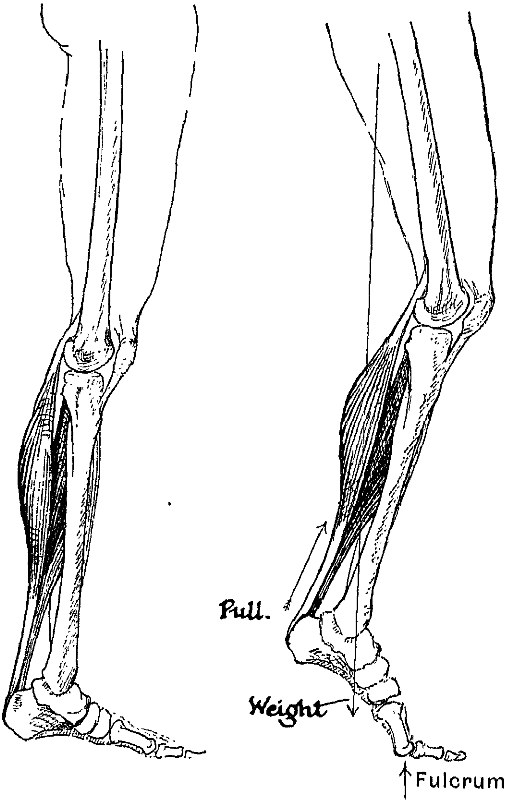

FIG. 17.

$$F \times d' = W \times d, \text{ or } \frac{W}{F} = \frac{d'}{d}.$$

The great convenience of a machine working at large mechanical disadvantage is that the applied force only moves through a very small distance compared with the movement of the weight or resistance overcome. Hence, although the limbs in gymnastic evolutions are given wide excursions, the contracting muscles which actuate the tightening tendons stay very much in their places.

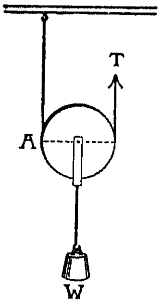

FIG. 18.

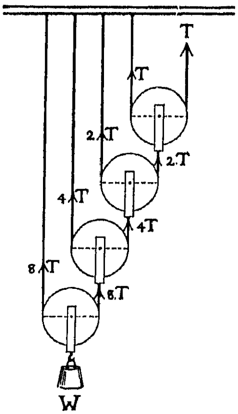

FIG. 19.

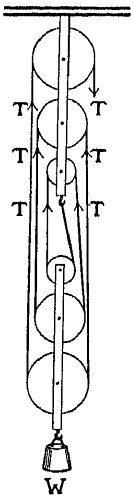

FIG. 20.

A pulley consists of a wheel mounted on a shaft and carrying a rope which serves as connection between the force applied and the weight lifted. When the pulley is fixed so that it is only free to rotate about an axis through its centre, there is no mechanical advantage. A single pulley used as shown in Fig. 18, however, gives a mechanical advantage of two. If T is the upward force or tension in the rope required to balance W, then since the moments about A are equal and opposite,

$$T \times \text{diameter} = W \times \text{radius}, \quad \textit{i.e.}, \ \frac{W}{T} = 2.$$

By combining a number of these pulleys, we may get still more mechanical advantage; it is assumed here that the tension of the rope is the same on both sides of a pulley. Hence, if we neglect the actual weight of the pulley and tackle, we see that by using four of such pulleys, as in Fig. 19, a single upward pull T would balance a weight $W = 16$ T.

Such an arrangement is very rarely used, although it has big mechanical advantage. The more usual way of coupling is as follows: the pulleys are fixed to an overhead beam or structure, as in Fig. 20, and the lower ones carrying the weight move up and down, the same rope going round each pulley. In this case, since the tension throughout the rope is the same, we have the upward pull of T in six ropes available to balance the weight W.

$$\therefore \text{ Mechanical advantage } \frac{W}{F} = 6.$$

In small working models these numerical relations do not hold because the weight of the movable pulleys is often nearly equal to the weight being lifted, but in actual practice this weight is generally only a small fraction of the real load.

HYDROSTATICS

The study of fluids includes that of liquids and gases. The difference between a solid and a liquid mass of the same material is that when it is solid, a body has shape, but when it is liquid, it has none; it takes the shape of any vessel into which it is poured. In some cases we get bodies which it is rather difficult to classify, because some solids have very little rigidity, for example putty or cobbler's wax, and some liquids such as treacle or pitch are very difficult to pour from one vessel to another; nevertheless the distinction is a real one, for when these substances are put to the test under very small forces, it is found that a very small force applied to a soft solid will not deform it even though applied for a very long time, but thick viscous liquids gradually give way if small forces are allowed to act upon them for a long time. Gases not only have no particular shape, but they have no particular volume; they have, in fact, no free surface; a gas, unlike a liquid, tends to fill any space into which it is introduced.

Though liquids and gases are much easier to deform than solids, yet any change of form is to some extent resisted, and this is called fluid friction or viscosity. We have seen that in attempting to move one solid surface over another, there is friction at the interface; in the motion of liquid through a pipe there is not only friction between the liquid and the inside surface of the pipe, but it is found that certain parts of the liquid are moving more quickly than others, and the friction that comes into play here is called viscosity. When water flows along a pipe the speed is greatest in the middle, and the water flows more and more slowly as we go from the central axis of the pipe to its inside surface; in fact, the liquid is at rest where it touches the surface of the pipe, except at very high speeds. Under these circumstances it is not surprising to find that when water is driven by the same force through pipes of different diameters, the speed with which it flows decreases very rapidly as the pipe becomes very narrow. This is generally expressed in the form of Poiseuille's law, which states that the amount passing along a pipe depends on the fourth power of the radius; one pipe double the diameter of another would let through sixteen times as much liquid in the same time, other conditions being the same. The blood vessels of the body vary very much in diameter, the capillaries are often only just large enough to allow the passage of red blood corpuscles, while some of the vessels near the heart form thick-walled tubes large enough to admit a finger or two. It will be easily understood, therefore, that while the blood is rapidly conveyed to distant parts of the body by the arterial system under the driving force of the heart, by the time it reaches the network of capillaries, fluid friction has slowed the rapidly moving blood stream to a very gentle trickle, quite insufficient in force to ensure its return to the heart were it not for some additional mechanism which helps the return of this blood by the veins.

Even though a liquid is as a whole at rest, it is subject to forces which must now be considered. The individual particles or molecules of the liquid are supposed to be in some sort of motion, but when we speak of a liquid as a whole being at rest, we mean that it is not obviously moving as it is when being stirred. Let us consider any small portion A of the liquid in Fig. 21. If it is at rest, it is not because there are no forces on it, for there are many reasons for believing that the molecules all round it are pressing on it in every direction normal to its surface and yet it stays at rest; the conclusion is that these fluid forces balance each other's effects; there is, for instance, no particular difficulty in seeing how the forces tending to push A from left to right balance those pushing it from right to left.

With regard to the up and down forces, the weight of the part of liquid inside A has got to be balanced somehow or other, or it would simply sink under the gravitational pull; but it is at rest, and so we might suggest that the upward fluid forces are greater than the downward fluid forces, and the difference is just equal to the weight of the liquid in A. We might go one stage further and say that whatever there might be inside the portion A, the action of the liquid in which it is immersed is such that it tends to push A upwards, *i.e.*, there is a vertical upthrust due to the liquid. This is a real enough thing as we can show by holding a cork under water and then letting it go; it happens in this case that the vertical upthrust is greater than the downward acting weight of the cork; the same experiment done with a lump of lead results in the lead sinking to the bottom of the vessel, but the upthrust is acting in this case too. If the cork and the lead were the same size, the vertical upthrust would be exactly equal in the two cases, yet one body rises and the other sinks. The recognition of this action of the liquid is generally referred to as the Archimedes principle, in honour of the discoverer. Archimedes showed that a body immersed in a liquid experiences a vertical upthrust equal to the weight of the liquid which by its immersion it displaces.

Now if a body immersed in a liquid experiences such an upthrust, we should expect it to lose weight,

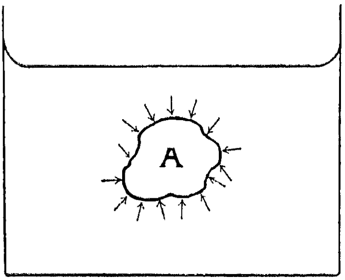

FIG. 21.

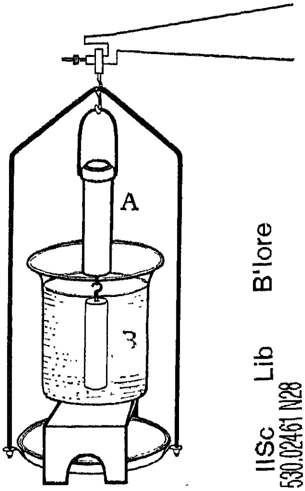

FIG. 22.

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and this is borne out by experiment; further, it is found that the weight it loses is just equal to the weight of the liquid it displaces. This is well shown by the following experiment: A hollow brass cylinder A, in Fig. 22, and a solid one B which just fits into the hollow one, are weighed in air by means of an ordinary balance. The solid cylinder hangs by a small thread to the bottom of the hollow one; while they are in this position the lower cylinder is immersed in a beaker of water, with the result that the balance is no longer equilibrated and it is found that the weights in the scale pan are heavier

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than the cylinders. If water is now poured into the hollow cylinder so as to fill it, the two sides will once again balance, and the volume of water required is exactly equal to the volume of the immersed cylinder, so that when we immerse any body in a liquid it loses weight by exactly the weight of the liquid it displaces. The specific gravity (S.G.) of a body is the weight of it compared with the weight of an equal volume of water. The most direct way of finding this is to weigh the body in air (W_1) and then in water (W_2); then $W_1 - W_2 =$ weight of an equal volume of water.

$$\text{Specific gravity} = \frac{W_1}{W_1 - W_2}.$$

In many cases this cannot be done because of chemical action between the solid and water. Recourse is then had to weighing the solid in some liquid which has no chemical action on the solid and then determining the specific gravity of the liquid itself. For liquids, determinations of their specific gravity are made by instruments known as hydrometers. The most practical form of hydrometer consists of a hollow glass stem with a graduated scale, with a glass bulb containing mercury attached to the end. The instrument, when put into a liquid, floats so that it is immersed up to a point on the scale, the reading of which gives the specific gravity of the liquid in which it is floating.

The specific gravity of a body must be carefully distinguished from its density, which is defined as the mass per unit volume of a body.

The buoyancy of the air is well shown in the case of a balloon. This consists of a large elastic envelope generally filled with hydrogen, though sometimes with helium, to which is fixed an under carriage for the passengers. If W_1 is the weight of the envelope and W_2 that of the under carriage, then if $W_1 + W_2$ is less than the weight of the air, W_3, which they displace, there will be an upward force on the balloon equal to $W_3 - (W_1 + W_2)$. This force causes the balloon to rise, and as it does so it reaches higher strata of air where the air is not so dense; on this account the buoyancy is reduced, but this is more than offset by the fact that, as the external pressure on the elastic envelope is reduced, the gas inside tends to stretch it. This increase in volume causes more air to be displaced and the balloon would rise still more quickly on this account. Most balloons are fitted with a contrivance which allows the containing gas to escape, so that the envelope is not stretched to a dangerous extent. With the advent of the aeroplane, this method of air travel is not often indulged in, but captive balloons tethered by ropes to the ground are still used for military purposes.

Since the weight of a body is proportional to its mass, the numbers which serve as a measure of density also serve as a measure of specific gravity. The density of water is 1 grm. per cub. cm., that of copper is 8·93 grm. per cub. cm., and since the weight of 1 cub. cm. of water is 1 grm., the specific gravity of copper is also given by the number 8·93; in this case it is only a number, but the density of a body has dimensions, grams per cubic centimetre or pounds per cubic foot, according to which set of units is being used.

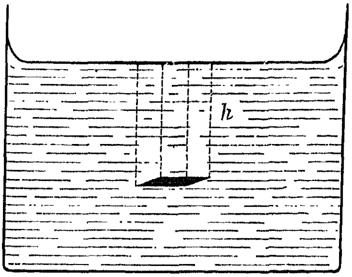

Fig. 23.

Let us now consider a liquid of density ρ in a vessel and imagine a horizontal area of 1 sq. cm., at a depth h cm. in it, as in Fig. 23. If we ignore for the moment the liquid beneath the area, then it is clear that there is a column of liquid above it whose weight is pressing down on the area. The volume of this column is $h \times 1$ cub. cm. and the weight of it will be $h\rho$ grm., and the force pressing down on the area will be $h\rho g$ dynes per sq. cm. This is known as the fluid pressure, or rather the pressure, *i.e.*, force per unit area. The liquid below the area presses upwards on the area with just this force and so balances the downward one.

It is customary to speak of the pressure at a point in a liquid, and by this is meant the force acting on a unit area surrounding the point in question in such a way that all parts of the area are at an equal depth in the liquid; if the area were placed vertically, it will be seen that the force of the liquid would be less at some places than at others. If an area A were placed with respect to the point, so that all parts of the area were at the same depth, *i.e.*, horizontally, we can see that the force acting on this area will be A times the pressure, *i.e.*, $Ah\rho g$ dynes, if A is in square centimetres, the other quantities being also expressed in C.G.S. units.

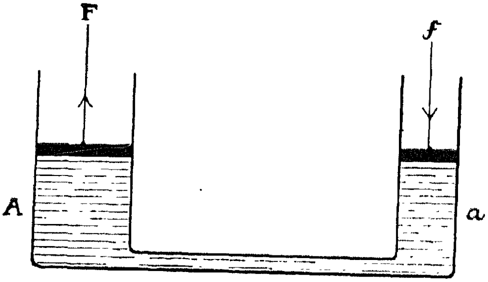

FIG. 24.

Experimentally these considerations are found to hold quite exactly, and the pressure at any depth in a liquid is found to depend only on the quantities which have been specified.

Further, it is found that if any external pressure is applied to a liquid, it is transmitted equally in every direction throughout the liquid. This is well shown by an arrangement called the hydrostatic paradox; two vertical tubes of cross-sectional areas, a and A, are joined by a horizontal tube and the whole arrangement filled with a liquid, as in Fig. 24. Imagine pistons fitted at the tops of the two vertical tubes and a force f

applied to the smaller one. Since the area of section is a, this force will give rise to a pressure f/a and this will be transmitted equal in every direction through the rest of the liquid; on the under surface of the large piston the pressure will also be f/a, and therefore the force acting will be F where $F = \frac{f}{a} \times A$.

$$\therefore \frac{F}{f} = \frac{A}{a}.$$

It is thus seen that a very small force f could balance a large one F, and the arrangement resembles that of a lever having considerable mechanical advantage. Use is made of this in the hydraulic press invented by Bramah, which bears his name.

It should be noted here that liquids are practically incompressible, there being no sensible diminution in their volume when enormous pressures are brought to bear on them.

SURFACE TENSION.—The free surface of a liquid at rest is horizontal; if it were not we should, of course, expect the part at a higher level to flow towards that at a lower level; if, however, we look carefully at the level of water in a beaker, we see that where the water meets the glass it seems to be drawn up towards it. This effect is due to what is called surface tension. If we imagine the beaker to become very narrow, then there will be no level part in the middle, and in fact by giving water the chance of coming more and more under the influence of this force, it can be drawn a considerable way up a narrow hollow glass tube; this is often called capillary attraction and the tube is called a capillary tube. In this case the water will stand at a height h in the tube above the ordinary level of the water, as shown in Fig. 25. If T is the surface tension of the liquid, then we have to think of this force acting round the ring of water where it touches the glass, and the effect of this force is to pull the water up the capillary; it pulls the water up until the weight of the water in the capillary acting downward just balances this upward pull.

If γ is radius of the tube and ρ the density of the liquid, we have—

$$T \times 2\pi\gamma = \pi\gamma^2 h\rho g.$$

$$\therefore\ T = \frac{\gamma h\rho g}{2} \text{ dynes per cm.}$$

For pure water the value of T is about 72 dynes per cm. Observation shows that in the case of mercury it is not pulled up inside a capillary but pushed downwards; the surface of the mercury would be as shown in Fig. 26. In such cases the liquid is said not to wet the glass, though it still has a surface tension. The fact that mercury stands lower in a capillary tube than in the liquid outside has to be borne in mind in making a barometer; with a wide tube the extent of the drag down is very small and is referred to as the meniscus correction.

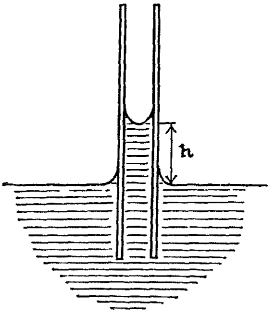

FIG. 25.

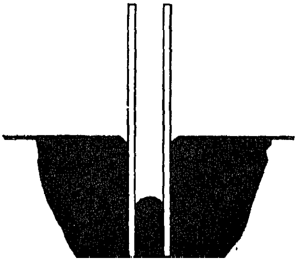

FIG. 26.

All liquids exhibit surface tension under appropriate conditions, and it is due to this force operating that water drops are formed, the force acting in such a way as to make a given volume of water have the least possible surface, *i.e.*, a sphere. When raindrops are formed they usually start by being very minute spherical drops, so small that they are spoken of as a mist or fog;

but where there are many of them in a small space, they coalesce to form bigger drops which still remain spherical. As they get bigger they begin to move downwards with appreciable speed, and owing to the resistance which they meet as they fall through the air, they are deformed and get elongated. Many different forms are possible for a liquid when under the action of surface tension, as may be seen in soap bubbles, but here again the spherical bubble is the most stable form.

Atmospheric Pressure.—The atmosphere of the earth surrounds it and extends upwards to a great height, although it gets appreciably less dense as the distance from the earth's surface increases. This belt of air exerts a pressure in the same way as a liquid does, but the ordinary expression giving the pressure at a depth ($p = \rho gh$) in a homogeneous liquid does not hold in this case, because the vertical gradient of density in the atmosphere is comparatively steep.

The fact that the atmosphere exerts a pressure was first shown by Torricelli, who in 1643 took a long glass tube sealed at one end, filled it with mercury and inverted it over a bowl of mercury; the result was that some mercury stayed in the tube and a column of mercury about 30 in. high stood above the level of the mercury in the outside vessel. The effect was rightly attributed by Torricelli to the action of the atmosphere pressing on the mercury and thus supporting the column within the tube. The space at the top of the tube contains no air and is often known as the Torricellian vacuum. The fact that the mercury column only reached up to a certain height indicated that the atmospheric pressure could only support a certain weight of mercury.

A few years after this discovery, Pascal showed that when a Torricelli tube was carried up a mountain the mercury stood at a lower level, and he inferred that this was due to a real diminution in the pressure as one ascended into the earth's atmosphere. This is a fact keenly realised by airmen flying at great heights; the discomforts of a freezing atmosphere are doubled by the extra effort involved in living in air which gets poorer in its supply of oxygen.

The air over the earth is rarely stationary; there are the big displacements caused by prevalent winds which may come from the same quarter for days, there are big upward movements of the air which are the precursors of rain, there are cyclonic states in which, over localised stretches, the atmosphere is rotating as well as moving in a definite direction as a whole, and there are the other quieter movements which result in the piling up of the air leading to the spells of quiet, fine weather that we call anti-cyclones.

If the height of the mercury standing in the Torricelli tube be watched, it will be found day by day to alter, sometimes rising, sometimes falling. The height is proportional to the pressure of the atmosphere, and the apparatus when used for this purpose is called a barometer. Care has to be taken that the last traces of air are got out of the space above the mercury, otherwise this exerts a pressure downwards and the mercury stands too low.

A wide tube should be used, otherwise the surface tension (p. 34) causes the mercury to be pulled slightly down the tube. A brass scale with vernier attachment makes it possible to read a barometer accurately to about one hundredth of a millimetre. In order that the observed height should serve as a reliable measure of the pressure of the atmosphere, we have to apply some corrections; the following are mentioned in their order of importance. Barometric readings are reduced to what they would be if the temperature were 0° C.

1. TEMPERATURE.—Mercury expands when it is heated, so if the level is at a height h when the temperature is $t°$, the height h_0 at a temperature of 0° C. would be less than this, and the relation between the two is given by

$$h_0 = h(1 - \beta t),$$

where β is the coefficient of expansion of mercury and has the value ·000181, and t is the temperature centigrade.

2. Temperature.—The brass scale expands when it is heated, and this makes the reading h at a temperature t a lower one on the scale than if the reading had been made at 0° C. In this case

$$h_0 = h(1 + \alpha t),$$

where α is the linear coefficient of expansion of brass and has the value ·000019. Combining 1 and 2, we may write for the corrected height—

$$h_0 = h(1 + (\alpha - \beta)t).$$

The reader will no doubt on first reading find some difficulty in these expressions if he knows nothing of the subject of temperature measurement or of expansion coefficients. These matters are dealt with in Chapter II.

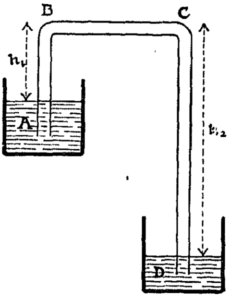

Fig. 27.

3. Vapour Pressure. — Here again we have to deal with a phenomenon which comes in for better treatment in Chapter II. But we may say here that the space above the mercury column contains some mercury in the form of vapour; this exerts a downward pressure on the mercury, and the effect gets more pronounced the higher the temperature.

4. Sea-level Correction.—Meteorological observations at different heights above sea-level are brought to the same level by using a correction amounting to + ·13 mm. per 1,000 metres above sea-level.

5. Capillarity.—The surface tension of the mercury tends to pull the mercury down the tube slightly. For a tube 2 cm. diameter, the amount is very small and the correction is about ·02 mm. to be added to the actual reading.

The device known as the siphon depends on the pressure of the air for its action. It consists of a tube full of water connecting two vessels which are at differen levels, as shown in Fig. 27. When in action, wate flows from one vessel to the other in spite of the fac that it has to go vertically upwards over one part o its path. The reason for the direction of flow may b seen from the diagram by considering the pressures a B and C. Let P = atmospheric pressure. The pressur at B is less than that at the level A and is given by $P - h_1\rho g$, where ρ is the density of the liquid. The pressure at C is similarly $P - h_2\rho g$.

Therefore the difference of pressure between B and C is $\rho g(h_2 - h_1)$. The bigger the difference $(h_2 - h_1)$, the greater the speed of flow.

A suction pump depends on the pressure of the air for its action. A vertical pipe has its lower end in water; the upper end is fitted with a piston and valve, as shown in Fig. 28. When the piston moves down it shuts the valve B and opens A, thus letting water into the space above, from which it goes out by the outlet D. On the upstroke A closes, B opens, thus allowing water to enter from the supply at C.

A force pump is used when water under pressure is wanted, as in a fire-engine. The action can readily be understood by reference to Fig. 29. When the piston moves down it shuts the valve B and opens A by virtue of the compression it produces on the water in the cylinder. On the upstroke A closes and B opens, thus allowing water to enter the cylinder again.

It will be readily seen that water will not rise from C up into the pump in either of these cases if the height BC exceeds that of the water barometer.

Although we may look upon liquids as practically incompressible, this is far from the case with gases, for quite small changes in pressure can cause appreciable changes in their volume. Robert Boyle (1627-91) found that a very simple law expresses the relation between the pressure and volume of a gas. Experimenting with air, he found that if he doubled the pressure on air, enclosed in a tube, the volume was halved; by doubling the pressure again, the volume was again halved, and so on. He states that the volume of a gas varies inversely as the pressure to which it is subjected, provided the temperature remains constant. This latter is a most important point, because if the temperature does not remain constant when the pressure changes, the simple relation does not hold. There are two ways in which a gas can be compressed or expanded: we may do it isothermally, *i.e.*, so that no change of temperature occurs, or we may do it adiabatically, *i.e.*, so that no heat goes into or out of the gas during the change in pressure. If we do it in the latter way, we find that suddenly increasing the pressure on a gas warms it, and suddenly decreasing the pressure cools it. Later it will be seen that when a gas is warmed it expands, and when it is cooled it contracts, so that it is easy to see that Boyle's law does not hold unless the temperature of the gas remains constant.

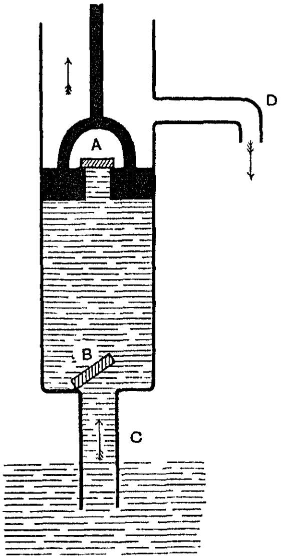

Fig. 28.

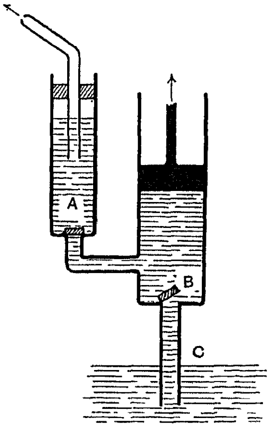

Fig. 29.

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We shall see later that air may be liquefied, and lik other liquids it is practically incompressible ; yet in th gaseous form it is easily compressed. We look upo the gaseous air and the liquid air as being made up o the same molecules, and we may at once questio whether the gaseous molecules are really compressibl and the liquid ones not. There are many difficultie in such a view, and the view is rather that in a ga the spaces between the molecules are much bigger tha in the liquid state, and that what we do in compressin a gas is merely to bring them nearer together withou in any way compressing the actual molecules. In the air about us the volume occupied by the molecules is only a very small part of the molecular interspaces, but in very high compression this is no longer the case, and under these conditions we find deviations from Boyle's law.

Diffusion.—Although there does not appear to be very much naked eye evidence for it, the view is held that solids, liquids, and gases are in a state of internal movement. That is to say, that the molecules are moving about to some extent. The extent of this movement in most solids is extremely small ; a piece of lead placed upon a piece of gold will remain in the same state for years without any apparent change, but Roberts Austen, for some years Master of the Mint, was able to show by very careful analysis that some lead made its way over the border-line into the gold and some gold got into the lead. Such a process is called diffusion. In liquids it is much more easily shown ; if a tall glass jar is filled to the half-way mark with a solution of copper sulphate and then water very carefully poured in to fill the jar, the line of demarcation is quite sharp between the blue solution of dense copper sulphate and the less dense water on top of it. If the fluids in the jar are kept quite undisturbed by any temperature changes, it is seen in the course of time that the sharp line between them becomes blurred, and this is due to some of the molecules of the sulphate diffusing upwards into the water, and some water diffusing down into the sulphate.

Graham made an elaborate study of the diffusion of liquids, and he came to the conclusion that liquids which conducted electricity well were more rapid diffusers than those that conducted it badly. He also showed that—

1. Solutions of the same strength of different substances diffuse with different velocities.
2. Solutions of the same salt of different strengths diffuse with velocities proportional to the strength.
3. The rate of diffusion increases with temperature.
4. The proportion of two salts in a mixture can be altered by diffusion.

Graham went farther than this and studied the way in which liquids diffuse through membranes; these membranes were either animal membranes or parchment. He found that some liquids could diffuse through such membranes, but that others would not; the former he called crystalloids and the latter colloids. If there were a mixture of the two, then the crystalloid could be separated from the colloid by letting it diffuse through a membrane which stopped the colloid; such a process he called dialysis and is of great practical importance.

Crystalloids include soluble salts and the acids. They have the following properties :—

1. They lower the vapour pressure of the liquid in which they are dissolved.
2. They lower the freezing point of the liquid in which they are dissolved.
3. They raise the boiling point of the liquid in which they are dissolved.

Typical examples of colloids are the gums, starch, gelatin. When they are put into water they have not got the properties 1, 2, and 3 mentioned above; their inability to diffuse through membranes is supposed to be owing to the largeness of their molecules.

Membranes can now be prepared which allow water to pass through them, but do not allow crystalloids to do so; these are called semi-permeable membranes. These membranes are rather difficult to prepare and require very delicate handling. One which has been largely used is the gelatinous precipitate of ferrocyanide of copper, produced when copper sulphate and potassium ferrocyanide come into contact. This precipitate was formed by Pfeiffer in the walls of a porous pot fitted with a pressure gauge, and he found that if the pot were filled with salt solution and then put in pure water, water flowed into the pot and compressed the air in the gauge. When a certain pressure was reached, no more water flowed into the pot.

If we started with big enough pressure inside the porous pot, no water at all would go into the salt solution, and the pressure needed to produce this state of affairs is called the osmotic pressure of the solution. Pfeiffer found that for dilute solutions which do not conduct electricity, the osmotic pressure is equal to the gaseous pressure which would be exerted by the molecules of the salt, if these were in the gaseous state and occupying a volume equal to that of the solvent in which the salt is dissolved and at the same temperature. Solutions which have the same osmotic pressure are called isotonic solutions; a solution called physiological saline is isotonic with blood, and does not cause either shrinkage or swelling of red blood corpuscles, because there is no tendency for fluid to pass either way.

The soft tissues of the body consist of collections of cells which are permeated by blood vessels, capillaries, lymph vessels, nerves, etc., between which there is a continuous interplay of fluid action; whether water penetrates into cells depends on a variety of conditions, one of which is the osmotic pressure of the contents of the cell compared with that of the fluid surrounding it.

Diffusion in gases is a very much quicker process than in liquids ; this is attributed to the much greater speeds of the molecules. In air the oxygen and nitrogen molecules are going about in every direction at an average speed of about 480 metres per sec., and although the numerous collisions which take place hinder the process of diffusion, this is nevertheless a very rapid process.

CHAPTER II

HEAT

Heat is said to be a mode of motion. The kind of motion that we think of in this case is not a motion that is obvious to the naked eye, for when heat goes from one body to another there is no motion that we can easily see in the process, and yet there is much evidence that some kind of motion is involved. Heat comes to us from the sun; we look upon the sun as a white hot body from which heat is streaming away into space, some of it reaching the earth and warming things that it strikes. The motion that we think of at the sun itself is a violent motion of the molecules or atoms present in the sun and its atmosphere; now this kind of motion gives rise to another kind rather more difficult to visualise. Between the earth and the sun is space practically devoid of matter, but it is customary to say that a medium called the æther fills space. Now when molecules are in such a state of motion that they are capable of giving out heat, we say that they radiate heat, and this they do by sending out some sort of motion into the æther. This we now call radiant heat, and this kind of motion is quite different from the molecular movement that gives rise to it, and is in some respects similar to the wave motion on the surface of the sea. So we have at least two kinds of motion to consider in heat, according to whether we are thinking of material things or of the all-pervading æther.

Experimentally it is found that heat tends to flow from a hot body to a colder one, and in order to define the degree of hotness or coldness of bodies, we invoke the term temperature.

A very careful distinction must be kept between temperature and heat. As we have said, heat tends to flow from the body at the higher temperature to another at a lower temperature, in spite of the fact that the one at the lower temperature might actually contain more heat than the other one. There is far less heat in a lighted match than in a bucket of water, yet heat quickly passes from the match when it is dropped in the water.

The instrument most used to measure temperature is the mercury-in-glass thermometer. One of the earliest was invented by Fahrenheit, and this is still used domestically and in medicine in this country. A capillary tube has a thin bulb blown at one end and clean mercury is run into it, boiled to expel the air and then the capillary is sealed off. The thermometer has two so-called fixed points which are determined in the following way: the thermometer is first put in melting ice and the position of the mercury thread marked on the capillary; it is then put in the steam from boiling water and the position of the thread marked; the two points are called the freezing and boiling points and are marked 32° and 212° F. The temperature of the steam from boiling water depends on the barometric pressure, so it is necessary to fix upon a standard pressure to work by, so that the boiling point determination can be compared at any time, whatever the barometer may read. The standard pressure chosen is 76 cm. of mercury.

Two other mercury thermometers have been constructed with different scale readings. One due to Celcius called the centigrade and the other due to Réamur. The fixed points on these are 0° and 100° C. and 0° and 80° R.

We must be able to correct any temperature on one of these thermometers to the reading which would be given if this same temperature had been observed with another kind of thermometer. For instance, say that we read a temperature of 98·4° F. (normal body temperature). This means that the mercury thread has gone 66·4 scale divisions above the freezing point. It will be seen from Fig. 30 that 180 scale divisions of this thermometer are equivalent to only 100 divisions on the centigrade scale, so that these 66·4 scale divisions would be equivalent to $66{\cdot}4 \times \frac{100}{180}$, *i.e.*, 36·9 scale divisions above the freezing point in the centigrade thermometer,

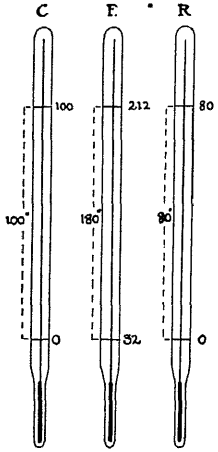


FIG. 30.

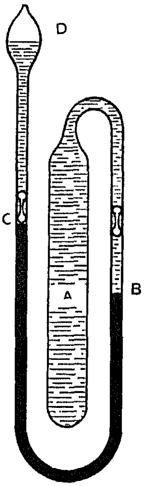


FIG. 31.

and since this is 0° C., the actual temperature will be 36·9° C.

When taking the temperature of a patient, it is essential that a proper reading can be got even though the thermometer is removed from the mouth or other region of the body where the temperature is taken. An ordinary thermometer would be of no use in this case, because the mercury would at once begin to move down the capillary tube. In the clinical thermometer this is avoided by having a small kink just where the bulb joins the capillary. When removed from the patient, the mercury immediately breaks off at this kink, leaving the thread in the position corresponding to the actual temperature to be measured.

For daily observations connected with the weather we want to know the maximum and minimum temperatures, and the special device adopted for this purpose is called a maximum and minimum thermometer. As shown in Fig. 31, this consists of a long bulb A filled with alcohol connected by a capillary tube with another bulb D about half full of alcohol. A mercury thread occupies a part of the connecting capillary tube. The position of this thread is conditioned by the expansion or contraction of the alcohol in A. If the temperature falls, the mercury in B rises and in C falls, and vice versa; small rods of glass or steel float on the surface of the mercury thread and get left behind at the farthest points, *i.e.*, at the minimum and maximum temperatures. Part of the daily routine of such observation work is the re-setting of the indicators.

The calibration of a thermometer is a necessary preliminary to any accurate work with it. This consists in finding whether there is any appreciable error in the so-called fixed point of the thermometer. When this has been done, errors at intermediate readings are usually found by simple graphical methods.

THE MEASUREMENT OF HEAT.—The unit of heat is called the calorie, and is the amount of heat required to raise the temperature of 1 grm. of water through 1° C. When measurements are very carefully made, it is found that 1 grm. of water does not require exactly the same amount of heat to heat it from 0° to 1° C. as it does from 1° to 2°, and so on. This quantity actually gets very slightly less as the temperature of the water is raised to 37° C., and then gets slightly greater again up to 100° C. It is not a very big difference throughout this whole range of temperature, and need not be taken into account except in very accurate work.

When we come to compare the amounts of heat required to raise the temperature of a gram of other substances, solid, liquid, or gaseous, through 1° C., we find very big differences in the amount required. This actual quantity is called the specific heat of the body.

The most direct method of measurement is called the method of mixtures and is carried out by means of an instrument called a calorimeter, which is nothing more than a thin copper pot. Suppose that the calorimeter contains some water at room temperature and we then put into it a piece of metal which has been warmed to a higher temperature than the water. Heat at once flows from the hot metal to the cooler water + calorimeter until the whole system reaches the same temperature. If we assume that no heat is actually lost in this operation—though some is always lost by cooling to the surroundings and can be allowed for—we can find out how much heat has been given out by the metal in cooling down through a certain range of temperature, and we assume that exactly this same amount would have been required to raise it through this same range of temperature. If we divide this quantity by the mass of the body multiplied by the range of temperature, we get the amount of heat required to raise unit mass of the body through 1° C., *i.e.*, its specific heat. It is clear that the calorimeter as well as the water in it is warmed by the hot body put into it, and this has to be taken into account when we actually balance the amount of heat given out by the metal against the amount received by the water. Other details connected with this kind of measurement are best dealt with in actual practical work.

The higher the atomic weight, the less the specific heat; this affords one reason for the use of mercury in thermometers; it has a low specific heat, and therefore a small amount of heat is required to raise the temperature of the mercury in the bulb; besides this, it is a liquid which can easily be prepared very pure, it does not wet the glass, and it has a considerable degree of expansion. Mercury freezes at about $-39°$ C., so that for lower temperatures alcohol is generally used; this has a freezing point of $-130°$ C. Mercury boils at 357° C., and beyond such temperatures it is usual to employ electrical thermometers, the mode of action of which will be discussed in a later section.

Dulong and Petit, two French physicists, noticed a very interesting connection between specific heats and atomic weights; the product of these two quantities gives a numerical value not very different from 6. A few examples are given in Table 1:—

TABLE 1.

Metal.	Specific Heat.	Atomic Weight.	Product. Sp. Ht. × At. Wt.
Aluminium .	·22	27·1	5·94
Iron . . .	·12	56	6·6
Copper . .	·09	63·6	5·9
Lead . . .	·03	206·9	6·2

EXPANSION.—Nearly all solids expand when they are warmed, though ice at 0° C. is a notable exception. If a metal rod is taken and fixed at one end and free play allowed to the other end, a measurable amount of expansion is produced by heating the rod through a moderate range of temperature. It is found that nearly all metals expand uniformly as they are heated, and this may be shown by measuring the expansion every few degrees, as the metal is raised in temperature, say, from 0° to 100° C.

It is found that the length l_t at any intermediate temperature t may be expressed in terms of the length l_0 at 0° C. as follows:—

$$l_t = l_0(1 + \alpha t).$$

α is called the coefficient of linear expansion.

A piece of copper 1 cm. long at 0° C. becomes 1·000015 cm. at 1° C., and the physical meaning attached to the quantity α is the amount by which a centimetre length of anything changes when raised through 1° C.

If we take a flat square sheet of copper measuring $l_0 \times l_0$ at 0° C. and raise its temperature to t° C., the area will be $l_t \times l_t$,

i.e., $$l_0 l_0(1+\alpha t)^2,$$

i.e., $$A_t = A_0(1+2\alpha t+\alpha^2 t^2) = A_0(1+2\alpha t).$$

When α is a small quantity (as it nearly always is) we can neglect α^2, and the coefficient of superficial expansion is just twice that of the coefficient of linear expansion. The term α^2 corresponds with the small shaded area in Fig. 32. In the same way, if we consider the expansion of a cubical block of copper, the volume V_t at any temperature can be expressed in terms of the volume at 0° C. as follows :—

$$V_t = V_0(1+3\alpha t).$$

Again neglecting the squared and higher terms of α, we get the result that the coefficient of volume expansion is three times that of the linear expansion.

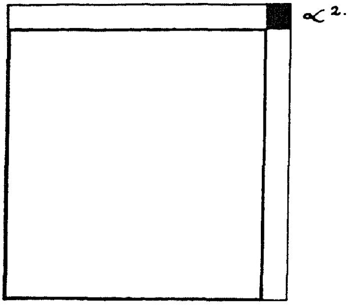


Fig. 32.

Some solids do not expand to quite the same extent in different directions, and in such cases we have to make use of the linear coefficients in directions which give unequal values. The expansion of solids has innumerable applications in everyday life—railway lines are laid with gaps between their ends to allow for the winter-summer range of temperature. Pendulum clocks would gain in winter and lose in summer were it not for the fact that their design includes temperature compensation, and the balance wheel of a watch is an ingenious example of combining two metals, which by their unequal expansion, will help to keep the effective radius unaltered, though the temperature ranges may be considerable.

The Forth Bridge, a massive steel structure 2,530 metres long, expands about 60 cm. in the temperature range of winter to summer.

The data in Table 2 give the coefficients of expansion of a number of solids.

TABLE 2.

Substance.	Coefficient of Linear Expansion.
Steel	·000012
Nickel	·000013
Brass	·000019
Lead	·000027
Zinc	·000029
Platinum	·0000086
Glass	·0000086

The fact that ordinary glass has the same coefficient of expansion as platinum allows of platinum being sealed into glass without fear of the latter cracking when there is a change of temperature.

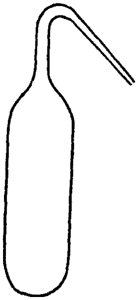


FIG. 33.

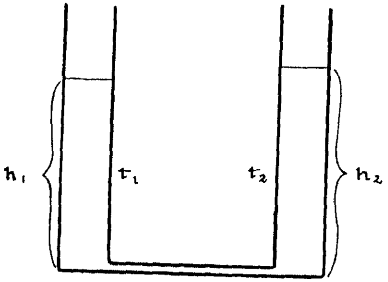


FIG. 34.

Quartz has an exceedingly small coefficient of expansion; in fact, it hardly expands at all, even when strongly heated.

Liquids generally expand more than solids do when they are warmed, but the measurement of this expansion is more difficult, because the vessel holding the liquid also expands. The first attempts on these lines gave what is called the " apparent coefficient of expansion," and a simple method of measuring this quantity for a liquid like mercury is to use a weight thermometer. This consists of a thin glass tube drawn out at one end and bent over, as shown in Fig. 33. This is filled with mercury at room temperature and weighed; let m_1 = mass of mercury in the tube. If the apparatus is warmed up through a range of temperature t, the mercury expands more than the glass and some will overflow; the tube remains full of mercury, but it will of course weigh less; let m_2 = mass of mercury now in the tube.

The apparent coefficient of cubical expansion of the mercury a will be

$$\frac{m_1 - m_2}{m_2 t}.$$

This may be proved as follows:—

The final mass m_2, if allowed to cool down to the first temperature, occupies a volume $\frac{m_2}{\rho}$, where ρ is the density of the mercury at that temperature; this volume expands to fill the weight thermometer at the higher temperature. So the increase in volume of a volume $\frac{m_2}{\rho}$ is $\frac{m_1 - m_2}{\rho}$. Ordinarily we should say the coefficient of expansion a is given by:—

$$\frac{\text{Increase in volume}}{\text{Original volume} \times \text{temperature change}}.$$

$$\therefore\ a = \frac{m_1 - m_2}{\rho} \div \frac{m_2}{\rho} \times t,\ \textit{i.e.},\ \frac{m_1 - m_2}{m_2 t}.$$

A method of finding the true or absolute expansion of mercury was devised by Dulong and Petit. This method consists in balancing a warm column of mercury against a cold one. Two vertical tubes (*vide* Fig. 34) are connected by a thin horizontal one. The mercury in one is kept at a temperature t_0, the other at a higher temperature t.

The pressure at any point in the mercury in the horizontal tube is the same, and the columns will be unequal in height, because as the mercury is warmed its density decreases and the heights of the balancing column will be inversely proportional to the density of the mercury in them, for

$$h_0 \rho_0 g = h \rho g \quad \therefore \frac{h}{h_0} = \frac{\rho_0}{\rho}.$$

If v_0 is the volume of unit mass of mercury at the temperature t_0, the density $\rho_0 = \frac{1}{v_0}$; and similarly,

$$\rho = \frac{1}{v} \quad \therefore \frac{h}{h_0} = \frac{v}{v_0}.$$

If β is the absolute coefficient of expansion of the mercury, we may write

$$v = v_0(1 + \beta t),$$

hence

$$\frac{h}{h_0} = \frac{v_0(1 + \beta t)}{v_0} = 1 + \beta t,$$

or

$$\beta t = \frac{h - h_0}{h_0}, \text{ i.e., } \beta = \frac{h - h_0}{h_0 t}.$$

Improvements in the means for measuring the actual heights of the mercury columns were made by Regnault, and since then the method has been put to use in finding the absolute expansion of a large number of liquids. The absolute coefficient of expansion of mercury was found by this means to be ·000181.

Water is very exceptional in the way in which it behaves when it is heated. If we take some water at 0° C. and warm it, it contracts until a temperature of 4° C. is reached, and then it expands on further warming; owing to this initial contraction, the density of water increases up to 4° C.; and this temperature is spoken of as the temperature of the maximum density of water, and this peculiar behaviour of water plays a very important part in the economy of Nature. A mass of water exposed to a cold wind will cool at its surface layers; as it cools down to 4° C., the water becomes denser than the water below it and sinks to the bottom, its place being taken by warmer water from below. This goes on until all the water is cooled down to 4° C.; on further cooling, the anomalous behaviour of water ensures that the surface layers as they cool still further remain on the top instead of sinking. The result is that although the surface layers may freeze, the water at a depth remains practically at 4° C. Of course under prolonged cooling at the surface, the ice may fall to many degrees below 0° C., and then heat becomes conducted upward to it from the warmer layers below and the whole temperature of the mass of water below diminishes.

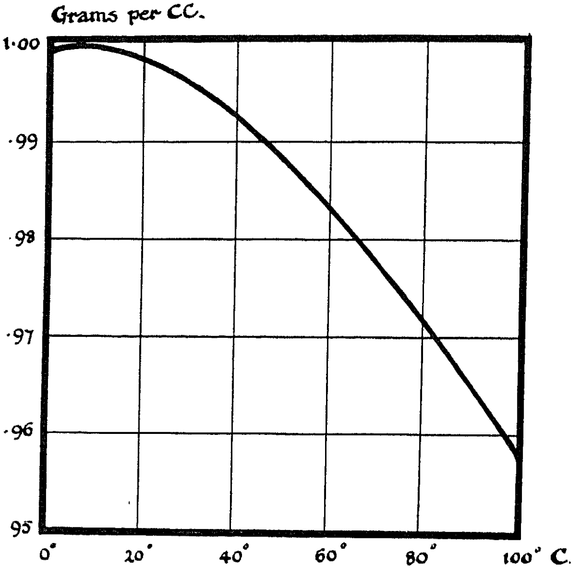


Fig. 35.

This important variation of the density of water as its temperature rises from 0° to 100° C. can be seen at a glance from the diagram in Fig. 35, the maximum value of the density being as stated at 4° C.

Gases expand and contract with heat more than do solids and liquids. It is remarkable that all gases expand practically to the same extent for the same range of temperature. This can be expressed in very similar form to that which we have employed with solids and liquids.

$$V_t = V_0(1 + at).$$

a is the coefficient of cubical expansion of the gas and has the value $\frac{1}{273}$. It must be understood that the above relation holds when measures are taken to keep the pressure constant, for should any variation of the pressure occur during the change in temperature, the volume would alter on this account also. The way in which the pressure is maintained constant in such measurement is as follows:—

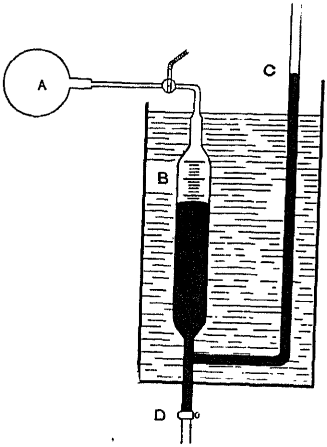


Fig. 36.

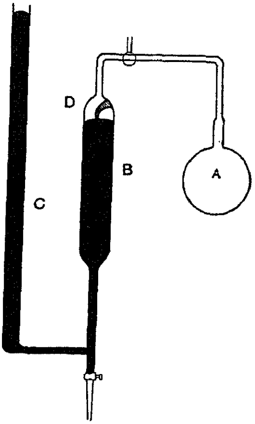


Fig. 37.

The air in a vessel A (Fig. 36) communicates with the air in another vessel B filled with mercury and carefully graduated; this vessel is provided with an outlet at D and a side limb C, from which the pressure can be read off.

When the air in A is heated, its volume increases and mercury is run out at D, and the level of the mercury in C adjusted so that the air in A is at exactly the same pressure as before. Working in this way over any desired range of temperature, it is found that

$$V_t = V_0(1 + at).$$

We may vary the experiment by seeing how the pressure in the vessel A would vary with temperature if we kept the volume constant. This is conveniently done as follows :—

The air in a vessel A (Fig. 37) at atmospheric pressure and 0° C. communicates with a vessel B which terminates in a vertical column C, mercury filling both of them. Suppose that the mercury in the connecting glass tube stands at **D**. If the air in A is heated, it tends to expand and push the mercury below **D**; but if the mercury at C is adjusted to stand higher and so increase the pressure on the gas, the mercury may be made to go back to the same point **D**; the volume of the air in A is thus kept constant.

Under these conditions it is found that an exactly similar sort of relation holds as in the previous experiment, and we may write

$$p_t = p_0(1 + \alpha t),$$

where p_t is the pressure at any temperature t in terms of the pressure at 0° C. As before, $\alpha = \frac{1}{273}$ and is the same for all gases.

If we examine the previous expression $V_t = V_0(1 + \alpha t)$, we see that if it were possible to cool the gas down to a temperature as low as 273 below 0° C., the equation would read

$$V_t = V_0\left(1 - \frac{273}{273}\right) = 0.$$

The interpretation of this would appear to be that the gas would occupy no volume at all at such a temperature, and an exactly similar line of reasoning would show that it would also exert no pressure at all. Experimentally it is found that gases do not remain gases at such low temperatures, but become liquids; but it is found extremely useful to make use of this impression of a lowest possible temperature, and $-273°$ C. is known as the absolute zero, and a scale of temperature called the absolute temperature has been founded with this zero as its lowest point. It will be seen that 0° C. will be 273° absolute temperature and 100° C. will be 373° absolute temperature, and so on.

It is customary to write T for the absolute temperature corresponding to any temperature t° C. We may re-write the equation

$$V_t = V_0(1+\alpha t) \text{ as } V_t = V_0\left(1+\frac{t}{273}\right) = V_0\left(\frac{273+t}{273}\right) = V_0 \cdot \frac{T}{T_0}$$

where T and T_0 are the absolute temperatures corresponding to t° C. and 0° C. We are now in a position to express the volume, pressure, and absolute temperature of a gas in a very convenient form.

Let us start with a gas having a volume V_1 at pressure p_1 and absolute temperature T_1.

Let the pressure be changed to p_2, the temperature being kept constant; hence the volume V_1 will change, let us say, to V. Applying Boyle's law, we may say

$$V = V_1\frac{p_1}{p_2}.$$

Let the temperature now change from T_1 to T_2, the pressure remaining constant at the value p_2; the volume will again change, let us say, to V_2.

Applying Charles's law, we may say

$$V_2 = V\frac{T_2}{T_1}.$$

Substituting the value found for V, we have

$$V_2 = V_1\frac{p_1}{p_2} \cdot \frac{T_2}{T_1};$$

or, re-writing it, we have

$$\frac{p_2V_2}{T_2} = \frac{p_1V_1}{T_1}.$$

Hence we see that starting with a known volume of gas at a known pressure and temperature, it may be put through any change of pressure or temperature without altering the value of $\frac{pV}{T}$. This important conclusion can be expressed by saying that

$$\frac{pV}{T} = \text{constant},$$

or $pV = RT$, where R is a constant, and this is called the gas constant and the equation is called the fundamental gas equation.

If V is the volume of unit mass of the gas, the value of R can be found; it varies, of course, with the nature of the gas.

As we have already mentioned, no gas will fulfil the conditions implied by the equation $\frac{pV}{T} = \text{constant}$ if the temperature is sufficiently lowered, for it is found that all gases can be liquefied. But the idea of such a gas has not been discarded, and it is called a perfect gas.

Carbon dioxide and sulphur dioxide are two gases which can be liquefied comparatively easily, but great difficulties had to be overcome before other gases like nitrogen and oxygen could be made to assume the liquid form. It was soon recognised that the things most likely to cause a gas to liquefy are to lower its temperature and at the same time to increase the pressure upon it; nevertheless it was found impossible to liquefy air when it was cooled down to a very low temperature, even though enormous pressures were brought to bear upon it. The secret of the process is that unless a certain lowness of temperature is reached, it is impossible to liquefy a gas by pressure alone, no matter how big a pressure is used. This particular temperature is called the critical temperature of the gas, and the pressure which is then necessary for liquefaction is called the critical pressure. If the gas is cooled down lower than its critical temperature, then some pressure less than the critical pressure will liquefy it. The data in Table 3 show how these quantities vary as we go from one gas to another :—

TABLE 3.

Gas.	Critical Temperature.	Critical Pressure.
		Atmospheres.
SO_2 . .	155·4°	78·9
CO_2 . .	31·1°	73
Air . .	−140°	39

The curves in Fig. 38 show the behaviour of CO_2 at different temperatures and pressures. Above 31·1° C. it is impossible to liquefy this gas. When carbon dioxide under pressure is allowed to escape fairly rapidly from the nozzle of a gas holder, it cools so rapidly as it escapes into the air that it is converted into solid carbon dioxide, sometimes called carbonic acid snow. This is used medically as a caustic agent; very cold things, like hot ones, will cause blistering when applied to the skin.

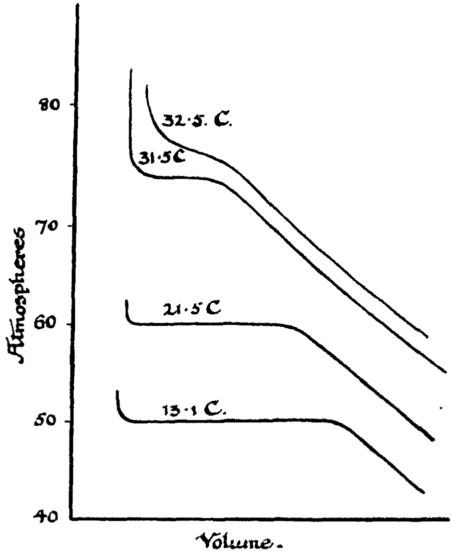


FIG. 38.

Every known gas has now been liquefied. The properties of ordinary solids at very low temperatures are found to change very much; for instance, the electrical resistance of copper is found to get less and less as the temperature is reduced, and at a temperature of a few degrees absolute its resistance has practically vanished.

A distinction is drawn between a gaseous substance according to whether it is above or below its critical temperature. If above it is called a gas, and if below it is called a vapour; in the latter case we have seen that additional pressure will always liquefy it. It is found that vapours also obey Boyle's law, provided the pressures applied to them are not very high.

We see, therefore, that substances can exist in one of three forms—the solid, liquid, or gaseous. It is found that when we try to change a solid into a liquid, some heat is necessary for the purpose, even though the temperature is not altered; similarly heat is required in order to change a liquid into a gas without any change of temperature. If we do the thing in the reverse manner, we find that when a gas changes into a liquid, heat is given out by the gas, and when the liquid becomes solid again, heat is given out. This heat is known as latent heat.

In changing 1 grm. of a solid into liquid at the same temperature, the heat required is called the latent heat of fusion.

In changing 1 grm. of a liquid into gas at the same temperature, the heat required is called the latent heat of vaporisation. These quantities are in many cases very significant ones indeed; for instance, in order to convert 1 grm. of ice at 0° C. into water at 0° C., we have to give the gram of ice 80 calories; 100 more calories would raise the gram of water to the boiling point; but if we want to convert the gram of boiling water into steam at 100° C., we must give it 536 more calories.

Reciprocally, when steam at 100° C. cools to water, each gram gives out 536 calories as it assumes liquid form; this explains how it is that a scald from steam is so much more serious than one caused by boiling water.

The experimental methods used for determining the latent heat of fusion or the latent heat of vaporisation is generally the method of mixtures already described in the case of specific heats. If a mass of melting ice is dropped into a calorimeter containing a mass of water at a known temperature, then from the resulting drop in temperature of the water the latent heat of fusion of the ice can be found. In a similar way by passing steam through a condenser immersed in water and equating the heat received by the condenser to that given out by the steam, the latent heat of vaporisation of the steam can be found.

The change in volume which occurs when ice is melted has been used by Bunsen in an instrument called Bunsen's ice calorimeter to serve as a measure of the heat given out by a hot body in cooling through a range of temperature and is used for measuring specific heats.

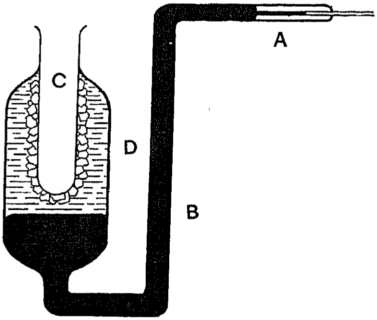


Fig. 39.

The instrument consists of a glass test tube C fixed into a bulb D, which connects via B with a horizontal capillary A. D is filled with well-boiled distilled water, except for the bottom part, which holds mercury, and this connects with the capillary A, which is marked off with a scale. A coating of ice is formed round the lower half of C by pouring alcohol cooled by a freezing mixture through it. The whole instrument is then packed round with crushed up ice and left, so that the temperature becomes 0° C. The method of finding the specific heat of a solid is to take a known mass of the body at a known temperature and drop it into C; this melts some of the ice round C; the decrease in volume is shown by the movement of the mercury in A. The amount of heat required to cause the mercury to move by a definite amount is first found by pouring a known amount of water at a definite temperature into C and noting the degree of movement of the mercury in A. Once calibrated in this way, the determination of specific heats is easy, but the initial work with the instrument is considerable.

Vapour Pressure.—If a vessel containing water is put under a bell jar at ordinary room temperature, some of the water goes out of the vessel into the air in the bell jar; it remains suspended in the air as water vapour. The amount so formed is independent of whether there is any air in the bell jar, and the pressure caused by this water vapour simply depends upon the temperature; the higher the temperature, the more water vapour goes into the space in the bell jar; the process by which the molecules of water leave the liquid and become vaporised is called evaporation.

If the arrangement is kept at a steady temperature, it is found that eventually no more water evaporates, and when the equilibrium condition is established, we say that the space or air in the bell jar is saturated with water vapour, or in other words, the water is exerting its maximum vapour pressure. If we have not given time for this to occur, the space is unsaturated.

The vapour pressure of water vapour is not very big at ordinary temperatures, as may be seen from the data in Table 4:—

Table 4.

Pressure of Water Vapour in Mms. of Mercury

2° C.	5·29	12° C.	10·51
4° C.	6·10	14° C.	11·98
6° C.	7·01	16° C.	13·62
8° C.	8·04	18° C.	15·46
10° C.	9·20	20° C.	17·51

From the fact that the air in the bell jar gets saturated with water vapour, we might conclude that evaporation stops at this point; but it is more usual to think of this process being gradually balanced by the opposite process of condensation, which means that some of the water vapour returns to the water, *i.e.*, it condenses. When these two opposite processes just balance each other, we say that the space is saturated; if we try to get more moisture into it, it promptly condenses. The same effect would be produced if we slightly cooled the air in the bell jar, for the vapour pressure of the water is at once lowered and the excess of water vapour condenses as a mist or a cloud.

It is found that when any liquid evaporates, its temperature is lowered, and this can be very strikingly shown in the case of water by means of an instrument called a cryophorus, seen in Fig. 40. A and B are two glass bulbs containing water, which are connected by a piece of wide glass tubing; the air is pumped out of the apparatus just before it is sealed up, B is put into a freezing mixture, A remaining at room temperature. The water in B quickly freezes and its vapour pressure is therefore very much diminished; the vapour pressure in A now being much greater than in B, water vapour quickly passes from A to B; the result of this evaporation is to lower the temperature of the water in A. This goes on till the water in A freezes.

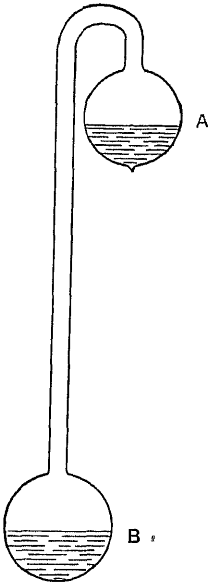


FIG. 40.

Our sense of bodily comfort depends to some extent upon the relative humidity of the air; by this we mean the ratio of the amount of water in the air to the amount that there would be in the air if it were saturated. If this ratio is a high one, it means that the air is nearly saturated.

This ratio may also be expressed as the ratio of :—

$$\frac{\text{The actual vapour pressure of the air}}{\text{Saturation pressure of the air}}.$$

By this we see that relative humidity does not refer to the actual amount of water vapour present in the air. It may easily happen that on a day in summer when the air feels pleasantly dry, it actually contains far more moisture than on a day in winter when with a low temperature the air feels as though it were laden with moisture, *i.e.*, practically saturated. In the former case the relative humidity will be low and in the latter case high, and it is the percentage of possible moisture that determines our ordinary sense of bodily comfort. Two methods are in common use for finding the relative humidity of the air, and this is called hygrometry.

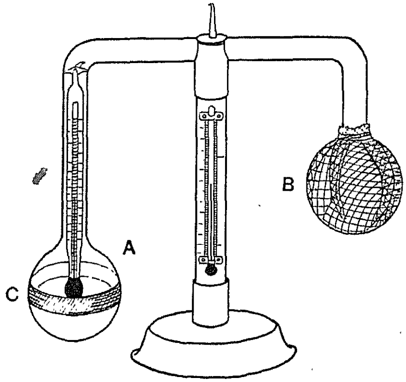


FIG. 41.

The most direct method is by means of the dew point hygrometer (Fig. 41). This consists of two bulbs connected together as in the cryophorus. The bulb A has a thermometer to register the temperature of the ether in it. The other is covered with muslin, and when ether is poured on to it the rapid evaporation of the ether lowers the temperature of B, and therefore in a short time that of A also. The air round the bulb A is thus cooled down, and if carried far enough we reach the state at which moisture becomes condensed from the air on to the bulb A; this is easily seen by noticing when a film of dew appears on a gilded band at C running round the bulb A. If T_1 is the temperature of the air and T_2 the temperature of the ether in A, and therefore that of the air at the moment that moisture is deposited, *i.e.*, the dew point, then from these two observations we get the relative humidity.

Example.—T_1 is 16° C. and T_2 10° C. when dew forms.

Saturation vapour pressure at 16° C. = 13·62 mm.
" " " 10° C. = 9·20 mm.

$$\text{Relative humidity} = \frac{9.2}{13{\cdot}62} = 67{\cdot}5 \text{ per cent.}$$

An indirect yet very quick method is to use an instrument called a wet and dry bulb thermometer.

Two thermometers are placed on a stand side by side, and one of them has the mercury bulb covered with a piece of wick that dips into a bottle of water; this wet bulb will have water continuously evaporating from its surface and will therefore read a lower temperature than the dry bulb thermometer. If the air is very dry, evaporation will be rapid, and a bigger difference of temperature is registered between the two thermometers than when the relative humidity is high.

With the aid of tables prepared by Glaisher, the relative humidity can be determined once the readings of the two thermometers have been made. In practice it is easier to get the dew point in this way than with the dew point hygrometer.

An instrument called a kata-thermometer has been devised by Leonard Hill which measures the cooling power of the air; the instrument gives the time required for a bulb containing a warm liquid to cool through a certain range of temperature. If the air has great cooling power, the time required for this will be short, and Hill has shown that the cooling power of the air has a great deal to do with the efficiency of manua labour in factory life.

Water vapour behaves as a comparatively light gas its density being about five-eighths that of air. It i given off by the lungs and skin of a normally health person at a rate which has been estimated at about 30 oz in twenty-four hours.

It is easy to see why, simply on account of humidit considerations, the air in crowded unventilated room soon becomes insufferable.

We can calculate the weight of a given volume o saturated air at a known temperature in the followin way :—

The air in a room measuring 6 × 10 × 12 metres a 18·5° C. and 756 mm. of mercury pressure is saturate with water vapour. In order to find the weight of ai plus water vapour, we may calculate each separately The weight of a litre of air at N.T.P. is 1·293 grm The water vapour in the room exerts a pressure o 16 mm. of Hg.

Therefore the pressure due to the air is 740 mm and its volume is 720 cubic metres, or $7{\cdot}2 \times 10^5$ litres.

Hence the weight of the air is

$$1{\cdot}293 \times 7{\cdot}2 \times 10^5 \times \frac{273}{291} \times \left(\frac{740}{760}\right) \text{ grm.} = 8{\cdot}5 \times 10^5 \text{ grm.}$$

The weight of water vapour is similarly given by

$$\tfrac{5}{8} \times 1{\cdot}293 \times 7{\cdot}2 \times 10^5 \times \frac{273}{291} \times \frac{16}{760} \text{ grm.} = 1{\cdot}1 \times 10^4 \text{ grm.}$$

$$\therefore \text{ Total weight} = {\cdot}851 \times 10^3 \text{ kg.}$$

The total weight of the air in the room is thus see to be nearly a ton, which seems at first sight a rathe big weight.

Thermal Conduction.—This means the conveyanc of heat from molecule to molecule of a substance. Th "mode of motion" here is thought to be a state o vibration, and by some process imperfectly understoo this state of vibration gets handed on from molecule t molecule, and heat is said to be conducted in this way from one part of a body to another, or indeed from one body to another.

A very good idea of the degree to which heat is conveyed by this means can be got by taking an iron rod about 1 sq. in. in cross section and about 2 ft. long; holes are drilled at A, B, C, and D to take thermometers, as in Fig. 42. One end of the rod is heated by a Bunsen burner. Several minutes will elapse before the thermometer A shows any rise. As time goes on each thermometer in turn indicates that it is receiving heat at a greater rate than the others. After such an experiment has been going about an hour, it will be found that D is rising at a greater rate than any of the others, and eventually the readings in A, B, C, and D become steady; the temperature of B is lower than A because the part of the bar between them is cooling to the air, B is higher than C for the same reason, and so on.

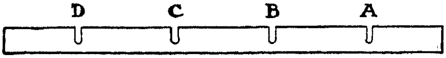


FIG. 42.

When different substances are put to the test, it is found that they conduct heat to very different extents. If we consider a slab of material of area A and thickness d, θ_1 and θ_2 being the temperatures of the two opposite faces (Fig. 43), then it is found experimentally that the heat passing through from one face to the other is directly proportional to the area A, to the temperature difference $\theta_1 - \theta_2$, to the time t for which the heat is allowed to flow, and inversely as the thickness d.

We may represent this as follows :—

$$Q = \frac{KA(\theta_1 - \theta_2)t}{d}.$$

K is a constant depending on the nature of the conducting substance and is called the thermal conductivity.

In Table 5 are given the values of K for a few solids :—

TABLE 5.

Substance.	Thermal Conductivity.
Copper	1
Brass	·30
Iron	·164
Ice	·0114
Sulphur	·0014

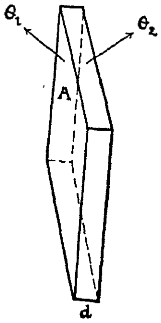


FIG. 43.

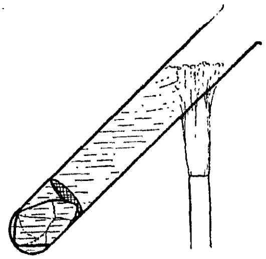


FIG. 44.

In very hot and very cold countries the therma conductivity of the materials of which the houses are made is a matter of the greatest importance. In the former case, massive walls of low conductivity prevent the interiors sharing in the baking heat of the exterior while in really low temperature regions, effective stoking within well-constructed buildings allows comfort within a few feet range of temperatures ranging down to 40 or 50 below 0° C.

The above expression allows us to calculate the amount of heat required to compensate for conduction losses.

Question.—Find the amount of heat lost per hour through each square metre of a wall 30 cm. thick, the temperature o one side being 15° C. and on the other −35° C., the thermal conductivity of the wall being ·018.

$$Q=\frac{KA(\theta_1-\theta_2)t}{d}$$

$$=\frac{\cdot 018\times 10^4\times 50\times 3600}{30}=1{,}000{,}000 \text{ calories approximately.}$$

Liquids and gases do not as a rule conduct heat well, in spite of the comparatively free motion of their constituent molecules. The low conductivity of water may be shown by trapping a piece of ice at the bottom of a test tube full of water and heating the water at the top by means of a Bunsen burner, as in Fig. 44. The water may boil at the top, yet ice remain at the bottom.

Pure thermal conduction in a gas is an even less efficient process than it is in liquids, but heat is actually transferred very effectually in gases and liquids by another process known as convection.

If we vary the test tube experiment by letting the ice float naturally to the top of the tube and apply the heat at the bottom, the ice will vanish in a very few minutes, and this is because heat has been convected through the water. When heat is applied at the bottom of the tube, the water as it is heated expands, therefore its density decreases and it tends to rise through the cooler parts, and in this way heat is rapidly spread through the contents of the test tube. This process is an even more rapid one in gases owing to the very big expansion which gases undergo when they are heated.

Circulating hot water systems depend for their action upon convection, hot water going from a boiler through a system of pipes and returning in a cooler and denser stream to feed the boiler once more.

There is a third way in which heat goes from one body to another, and this is by radiation.

We say that the sun radiates heat, and in order to understand this process of radiation, we have to introduce a different idea from that of conduction or of convection. Between the sun's atmosphere and our own is a vast space devoid of matter, yet heat passes freely across this gap, and we call this radiant heat or heat radiation. In order to make such a phenomenon more easily comprehensible, it has been assumed, as stated on p. 44, that space is really permeated by a medium called the æther, and that this æther is capable of being thrown into a state resembling vibration, though it is not thought that it actually vibrates in a mechanical way at all. The term æther vibration is often used to express this analogous action, but the term "periodic disturbance," though rather cumbersome, is free from the objection that can be raised to the term vibration in this connection.

Hot bodies have the power of causing disturbances to travel through this æther with the amazing speed of about 186,000 miles per sec. These heat radiations have not all the same characteristics, for it is found that when one body is hotter than another it may not only radiate more heat, but it radiates different heat and the difference we specify in terms of the wave-length of the disturbances.

By analogy with water waves we say that the distance between one point in space to the next in the line of travel of the heat, where a particular characteristic repeats itself, is the wave-length, and radiant heat in general consists of a vast number of disturbances of different wave-lengths. A very hot body will give out radiation consisting of many more different wave-lengths than a moderately warm one.

Bodies vary a great deal in their capacity for radiating heat. If we take two copper calorimeters of the same size, polish one brightly and cover the outside surface of the other with lampblack, fill them both with hot water, put a thermometer in each and take readings of the temperature from time to time, we find that the one covered with lampblack falls in temperature quicker than the other one. This is because the lampblack surface is a better radiator than the polished surface. It is also found that a good radiator of heat is also a good absorber, whereas a bad radiator is a bad absorber.

When radiant heat falls on a material body, it may be transmitted or it may be absorbed by the body. Bodies which allow radiant heat to penetrate them easily are called dia-thermanous bodies; those that do not are called a-thermanous.

Ordinary window glass is a substance that is dia-thermanous for some wave-lengths of radiant heat, but a-thermanous for others. Owing to this a glass-house acts as a sort of trap for radiant heat; the glass lets through nearly all the short wave-lengths of the sun's heat, but it absorbs nearly all the long wave-lengths given out by the objects within, so that the temperature inside rises considerably above that outside.

Water is a fairly a-thermanous body, and not much radiant heat gets through about a foot of water. Rock salt, on the other hand, is practically dia-thermanous for all wave-lengths of radiant heat. Both of these substances are transparent to light, so that from this it is obvious that it is impossible to predict the way in which a body behaves to heat radiation from a knowledge of its behaviour towards light.

The air inside an incubator is kept at a constant temperature by means of a device called a thermostat. This works in such a way that enough heat is given to the apparatus to raise it to the desired temperature and is then automatically cut off; when the temperature begins to fall owing to loss of heat to neighbouring bodies which are at a lower temperature, the heat supply is automatically restored and so on.

The human body represents one of the most amazing constant temperature systems. In spite of big extremes in the meteorological conditions, occupation, or states of activity of mind and body, the temperature in normal health departs very little from 98·4° F. So fine is this adjustment, that for people who habitually give this as their temperature, a variation of 1° on either side of it is looked upon as no longer a normal temperature. It is perhaps unnecessary to add that the three processes of conduction, convection, and radiation are ea concerned in this steady maintenance of temperature.

VACUUM VESSELS. When air was first liquefie there was great difficulty in keeping it in this stat because heat penetrated the walls of the vessel in whi it was kept.

Dewar invented what he called vacuum vessels, ar these were in use long before the so-called thermos flas were commercialised. The design of the Dewar vacuu vessel is so good that liquid air at a temperature 180 C. can be kept in a room at ordinary temperatu and yet evaporate so slowly that much of it will rema there for several days. This design aims at preventin any heat getting to the inside of the vessel and effected by having one glass vessel, spherical or cylin drical, sealed inside another and the space betwee them evacuated. The surface of the glass is silvered s as to reflect any radiant heat incident upon the oute vessel. Pure thermal conduction from the outside glas to the inside glass surface is entirely prevented b removing the air: this is done by a good vacuum pump

THE MECHANICAL EQUIVALENT OF HEAT.—Up t about the year 1798 the current theory as to the natur of heat was that it consisted of a fluid called caloric and a hot body was looked upon as one that containe a lot of caloric, whereas a cold one had little. Thoug there was no experimental proof of the existence of thi fluid, there was a reasonableness for it which kept i alive. But the experiments of Count Rumford in th year mentioned weakened to some extent the hold o the theory upon scientific people. Rumford, who wa engaged in the construction of guns, noticed that i the boring of a gun, a great amount of heat was generated he rightly attributed this to a direct conversion into heat o a part of the mechanical work which was being expended Sir Humphry Davy, a few years later, showed that b rubbing two pieces of ice together, he could melt them Both of these observations were difficult to reconcil with the caloric theory of heat, and this theory wa finally disproved by the experiments of Joule, who carried out a long series of most carefully planned experiments, which showed that by performing a certain amount of mechanical work, a definite amount of heat could be generated. Joule had a vessel containing water at a known temperature, in which a paddle wheel was rotated a certain number of times; this rotation was caused by two sets of weights in falling through a measured distance. These weights could be wound up again and let fall through the same distance, time after time, the total work done being easily calculated.

Joule proved that a given amount of mechanical work performed in this way gave rise to the production of an equally definite amount of heat. This equivalence is referred to as the mechanical equivalent of heat.

Joule found that

772 ft.-lbs. of work could raise 1 lb. of water through 1° F.

We may transpose this to the C.G.S. units as follows:—

772 ft.-lbs. can raise 1 lb. of water through 1° F.

$\therefore$ 772 ft.-grms. can raise 1 grm. of water through 5/9° C.

$\therefore$ $772 \times 30{\cdot}48 \times 981$ dyne cms. can raise 1 grm. through 5/9° C.

$\therefore$ $772 \times \frac{9}{5} \times 30{\cdot}48 \times 981$ ergs can raise 1 grm. through 1° C.,

i.e., $4{\cdot}15 \times 10^7$ ergs are equivalent to 1 calorie.

It is a remarkable tribute to the work of Joule that all the measurements that have since been done on this most important equivalence have confirmed Joule's claims and made hardly any appreciable alteration necessary to the numerical values he obtained. It is customary to take $4{\cdot}2 \times 10^7$ ergs as the value of the mechanical equivalent of heat.

On the basis of Joule's work, the first law of thermodynamics was established, and this law is generally stated in the following terms: "Heat and mechanical work are mutually convertible, and in any operation involving such conversion, $4{\cdot}18 \times 10^7$ ergs of mechanical work disappear for each calorie generated, or $4{\cdot}18 \times 10^7$

ergs of mechanical work appear for each calorie expended."

A simple type of laboratory experiment showing that heat can be generated when work is done is as follows:—

A cardboard tube about a yard long is fitted with two cork bungs; a handful of lead shot is put into the tube, its temperature having been carefully measured. The bungs are inserted and the tube is then inverted about fifty times, care being taken that the shot falls freely down the tube; the temperature will be found to have risen between 1° and 2° and, knowing the specific heat of lead, we can get the ratio between the work done and the heat generated; it is difficult to get a result of greater than 10 per cent. accuracy, but the experiment shows very simply the direct generation of heat by nothing else than the performance of a certain amount of mechanical work.

CHAPTER III

LIGHT

LIGHT is said to be a disturbance in the æther, and of such a kind that it causes the sensation of vision.

These disturbances are, in the first place, produced by material bodies which are generally, though not always, hot bodies, so that heat rays usually accompany light; but when necessary the heat can be absorbed or separated in some other way from the light.

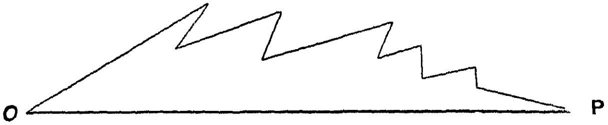


FIG. 45.

A body which emits light is called a self-luminous body; when this light falls upon other bodies it may make them luminous; in fact most of the objects around us in everyday life are recognisable owing to the light which they reflect or scatter.

The simplest self-luminous body is a point-source of light; this simply means a body so small that it can be imagined as a point showering out light equally in every direction.

Light is said to travel in straight lines. If O is a luminous source (Fig. 45) and P any point in the surrounding space, then the disturbances spreading out from O which reach P will have gone through points which lie successively on the line OP, and not by a zigzag path such as shown in Fig. 45. A flash of lightning might lead us to think that light does sometimes travel in this fasion, but in this case, of course, it is really the electrical discharge which is zigzag and the luminosity is a secondary feature of such a discharge. Exactly what kind of disturbance light is will be considered later, but it should not be imagined that when light goes from O to P anything goes along the line OP which is not actually shared by all similar lines originating at O. Light is a volume disturbance and is not restricted to definite lines, such as might be hastily assumed from diagrams showing its rectilinear propagation.

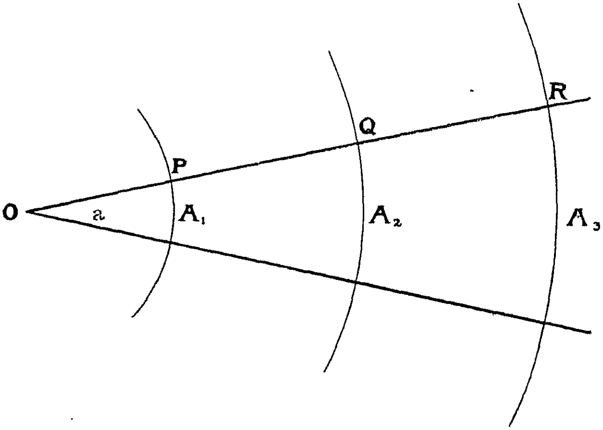


FIG. 46.

This fact will help us to understand how it is that the intensity of the light from a point-source O diminishes as we go away from the source. Let us start from such a source O (Fig. 46) and draw circles of radii r_1, r_2, r_3 such that $r_2 = 2r_1$ and $r_3 = 3r_1$ with O as centre. We shall assume that none of the light sent out from O is absorbed by the medium through which it passes. Consider the beam of light restricted tô an angle *a*. If A_1, A_2, A_3 are the areas of the circumscribing spheres having the radii r_1, r_2, r_3, then it is clear that all the light from O which falls on the area A_1 will also fall on A_2 and on A_3 in succession; but these areas will be illuminated to a less and less degree. If we regard the intensity of illumination as depending on the amount of light falling on a unit of area placed perpendicularly to the rays, then the intensity over these three areas will be proportional to $\frac{1}{A_1}, \frac{1}{A_2}, \frac{1}{A_3}$, and these in turn, since they are sections out of spheres of radii, r_1, r_2, r_3, will be proportional to $\frac{1}{{r_1}^2}, \frac{1}{{r_2}^2}, \frac{1}{{r_3}^2}$ since the area of a sphere is $4\pi r^2$.

Hence we reach the important conclusion that, when light issues from a point-source, the intensity of illumination in a non-absorbing space surrounding it, varies inversely as the square of the distance from the point-source; 1 ft. away, the intensity of illumination will be nine times what it is at a distance of 3 ft.

It is not a practicable matter to measure the actual amount of light given out by different luminous bodies, yet a comparison of their illuminating powers is often wanted.

For this purpose the light from any source may be compared with that given out by a standard source, which is called a standard candle. This is defined as a sperm candle, six of which weigh 1 lb. and such that in air it burns at the rate of 120 grains per sec.

In actual practice very little use is made of these standard candles, owing to their many inconveniences, and they have been replaced by the Harcourt Pentane lamp, which is equivalent to ten standard candles. The illuminating power of any source is expressed as so many candle-power. The unit of intensity of illumination is taken to be that which would be produced by a standard candle 1 metre away, and is referred to as a metre-candle.

Instruments used for the measurement of the candle-power of luminous sources and of the intensity of illumination which they produce are known as photometers.

A simple form of photometer is that devised by

Bunsen, called the grease spot photometer. A screen of paper has a central spot about $\frac{1}{2}$ in. in diameter, made by rubbing a little vaseline or similar substance into the paper; this makes the paper more transparent to light, so that the spot looks brighter than the rest of the paper when viewed by transmitted light, but darker than the paper when it is viewed by reflected light. Let us suppose that such a screen is now illuminated by two point-sources, one on each side of it, as shown in Fig. 47.

Let us suppose that the screen is viewed by an observer at A. Then if the screen is illuminated only by the source S_1, the spot will appear brighter than the screen, whereas if viewed when illuminated only by the source S_2, the spot will appear darker than the screen. We can arrange experimentally, however, that when the two sources of light are acting together, the spot appears of the same brightness as the screen, and can in fact hardly be distinguished from it. In this case the intensity of illumination over the screen is exactly the same from either source, and invoking the inverse square law we may say:—

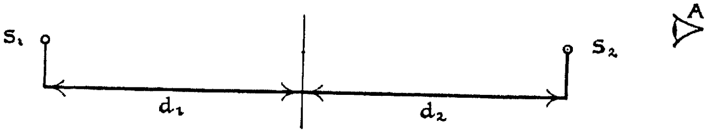


FIG. 47.

$$\frac{S_1}{d_1^2}=\frac{S_2}{d_2^2}.$$

If S_1 be a standard candle or its equivalent, we may in this way find the candle-power of the source S_2.

Rumford invented a shadow photometer which is sometimes used. A candle casts a shadow of a vertical rod on a screen; if another source of light be brought near, it will cast another shadow of the rod and will at the same time tend to eliminate the first shadow; by adjusting the sources of light, the two shadows can be brought close to each other and made to appear of the same depth ; the candle-powers of the two sources will bear to one another the inverse ratio of the squares of their distances from the screen.

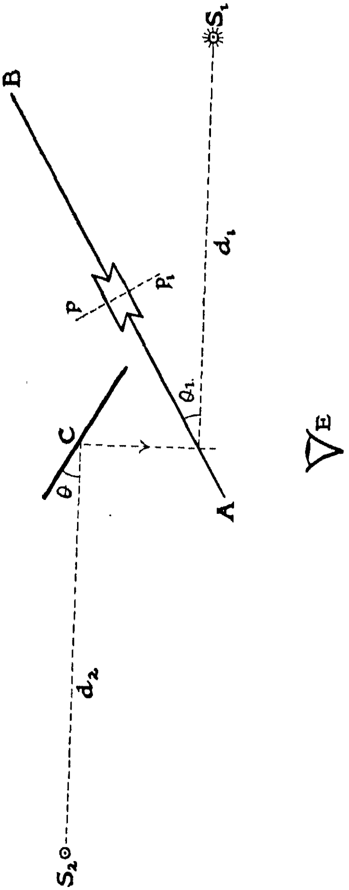


Fig. 48.

Joly has invented a simple photometer which consists of two slabs of solid paraffin set side by side, but separated by a sheet of metal. These slabs are very effective scatterers of light, and if a candle be set up about 1 ft. on one side of the photometer, one of the paraffin blocks appears to be well illuminated by it, the other remaining invisible; if another source of light be put on the other side, the corresponding block appears to light up, and once again it is only a matter of adjustment to make the two blocks appear equally lit, which makes it possible to compare the candle-power of the two sources.

Joly's photometer brings out particularly well the difficulty of comparing the candle-power of two sources which do not give out exactly the same kind of light. For instance, if a candle and the light from a small electric bulb are used, the paraffin block illuminated by the candle appears definitely pinker than the other, though the two blocks appear equally bright. This colour difference sometimes makes it very difficult to compare two luminous sources, but the difficulty is got rid of to a great extent by the Flicker photometer, which is devised so that the two surfaces being tested are presented to the eye intermittently; if they are not equally bright, a flicker can be detected, but if they are equally bright, even though they are not the same colour, no flicker is detected. This photometer is used in the following way: A circular disc (Fig. 48) having two sectors cut out of it is illuminated by a source S_1 and made to revolve in front of a screen C illuminated by a source S_2. AB is the disc mounted so as to rotate round the axis PP_1. An observer at E sees first a region A illuminated by S_1 and then one at C illuminated by S_2. When there is no flicker, the ordinary inverse square relation holds, provided the angles marked θ, θ_1 are equal.

Photometers are not only used to compare the candle-power of sources, but they can be used to find out the degree of absorption which light may undergo in passing through different media.

Optical media are classified as being transparent, translucent, or opaque. Transparent indicates that we may see clearly through such a medium, clearly enough to read print; typical examples are air, water, glass. A medium is translucent if it allows light to go into or through it and yet does not allow of clear vision through it; a sea mist, water containing a little milk or clay, and frosted glass are examples of translucent media. Opacity means that light cannot penetrate; all metallic bodies are opaque, but here it is sometimes a question of thickness, for gold may be beaten out so thin that green light is able to penetrate it to an appreciable extent.

Transparency does not mean that all the light incident penetrates; ordinary window glass absorbs a small percentage of the light incident on it. The percentage absorption can be determined very simply by photometry, as may be seen from the following example:—

A standard candle I is found to give illumination equal to a lamp L when the distances are 45 and 51 cm. respectively. Hence, if I and L measure their candle-powers—

$$\frac{I}{(45)^2}=\frac{L}{(51)^2}.$$

If a slab of glass is placed between the lamp and the screen, cutting down the illumination so that L has to be moved to a distance 48·6 cm. in order to balance I, the equation would now be—

$$\frac{I}{(45)^2}=\frac{L^1}{(48\cdot6)^2},$$

where L^1 is the effective candle-power of the lamp.

$$\therefore\ \frac{L}{(51)^2}=\frac{L^1}{(48\cdot6)^2},\ \textit{i.e.},\ L^1=90\text{ per cent. }L.$$

The slab has therefore absorbed 10 per cent. of the light incident upon it from the lamp.

Reflection and Refraction.—Very important differences are observed when light goes from one medium to another; the term optical medium is used to denote that a medium has optical properties which cannot be predicted from a knowledge of its other characteristics; it is a matter of experiment and measurement to determine them.

Say that a beam of light AB (Fig. 49) going through air falls on a flat sheet of glass; draw BO normal to the surface at B. Some of the light is reflected, and all that is "regularly" reflected will be found to be along BC, so that the angle of incidence i is equal to the angle of reflection, or $< ABO = < CBO$ and AB, BC lie in the

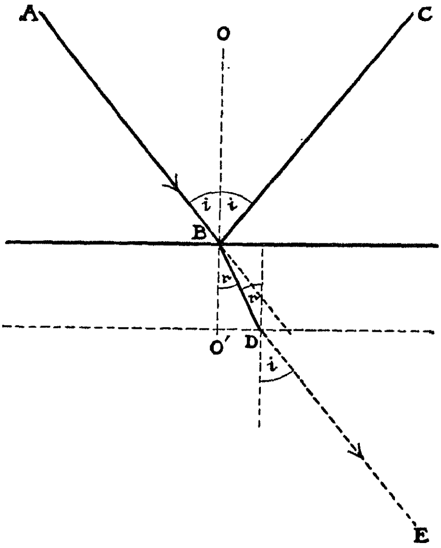


Fig. 49.

same plane (Snell's law). Some of the light goes into the glass, and in doing so is refracted, *i.e.*, bent out of its original direction. Experimentally it is found that the beam is bent in such a way that it gets nearer to the normal drawn from B in such a direction as BD. The angle DBO′ is called the angle of refraction r, and a certain relation has been found to exist between the angles i and r; this is expressed as follows:—

$$\frac{\text{Sin } i}{\text{Sin } r} = \text{a constant.}$$

This constant is called the index of refraction and is generally written μ. As in the case of reflection, the incident and refracted rays lie in the same plane (Snell's law).

The amount of light that is reflected from a surface depends upon the nature of the surface; highly polished metals are able to reflect nearly all of the light falling on them, but a piece of black paper reflects hardly any at all; most transparent bodies reflect very little, but the extent to which they do reflect light depends very largely upon the angle of incidence. For instance, in the case of the sheet of glass just considered, if the beam hits the glass nearly perpendicularly very little light is reflected, it nearly all goes through; but if the beam makes a big angle with the normal, *i.e.*, nearly grazing the glass, a very large percentage is reflected; the same is true for water, and the blaze of light from a river or wet road, with the sun low down on the horizon, is good proof of this.

The reflection that we have so far considered is sometimes called "regular" reflection; but there are important instances when light, incident at a definite angle upon a surface, appears to be reflected in every direction, by a sheet of white paper for instance. If a beam of light limited to quite a small angle falls on the paper, the latter is visible from practically any possible angle; the light is, in fact, scattered in every direction. This apparent contradiction to Snell's law, that the reflected and incident rays lie in the same plane, is due to the fact that the surface of the paper is not as even as it appears to be, and in fact magnification would show that it is extremely irregular; the result is that the incident beam is really incident at a large range of angles and correspondingly reflected over a large range of direction.

Referring once more to Fig. 49, observation shows that the refracted ray BD, on emerging from the glass into air, is deviated from its path; but in this instance it is bent away from the normal, taking the direction DE.

In the previous case we took μ to be the index of refraction of a ray of light, going from air to glass, *i.e.*,

$$\frac{\text{Sin } i}{\text{Sin } r}=\mu.$$

The same sort of relation will hold when it goes from glass to air; so that the emergent angle i will equal the incident angle i, *i.e.*, the ray DE is parallel to the ray AB; the effect of the sheet of glass has been not to alter entirely the direction of AB, but merely to give it a lateral shift; if the glass is very thin this is hardly appreciable, but it is easily measured when using a thick slab.

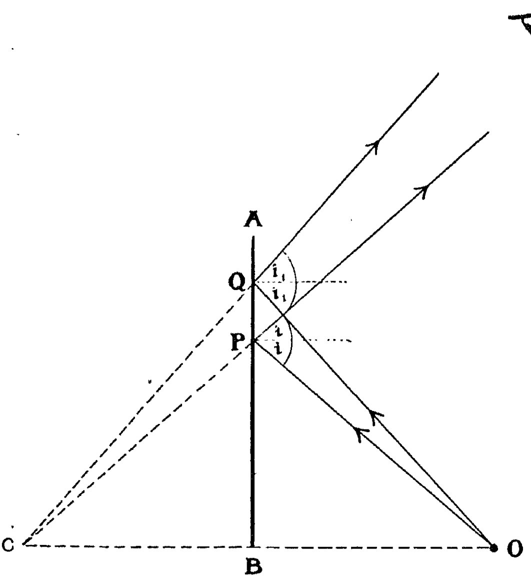


FIG. 50.

The two laws of reflection are applicable for the solution of problems involving the reflection of light from plane or curved surfaces. The case of a plane mirror will be considered first.

Let O be a luminous point in front of a plane mirror AB (Fig. 50). Draw any beam of rays OP, OQ, to meet the mirror. The ray OP incident at an angle i will be reflected at an equal angle; similarly for OQ, and since i' is greater than i, the beam leaves the mirror as a divergent beam, *i.e.*, it goes out into the air, widening as it does so. If an eye be placed in this direction, it sees an image of O at C. Exactly how this is brought about is mainly by physiological processes, because the divergent beam certainly does not go back along the paths PC, QC, the mirror actually letting no light go through it; yet an image of O is seen at C, and the position of C is exactly in the position which would be obtained if the reflected rays are traced back through the mirror. Such an image is called a virtual image, because it is not really there; but virtual images have a wide application in optics. The following rules for obtaining optical images will be found to be very useful:—

1. If a pencil of rays originating in a single point is made to converge to, or to appear to diverge from, some other point, the second point is called the optical image of the first.
2. If the rays actually pass through the second point, the image is said to be real.
3. If the rays do not actually pass through the second point, the image is called virtual.

An image, therefore, is the point where rays intersect or appear to intersect, and so at least two rays must be used in order to find the image of an object geometrically.

This holds when dealing with cases other than point objects; a point of an object and the corresponding point of its image are called conjugate points. It is a general principle in optics that, if a real image of a point O is formed at a second point I, then the image of an object placed at I would be seen at O.

A very important phenomenon occurs when light passes from one medium to another which is optically less dense. This phenomenon, which is known as total internal reflection, is well exhibited when light from a luminous object in water comes to the water-air interface. Let O in Fig. 50A be a luminous point under water;

then any ray such as OP when it emerges into the air above will be refracted *away* from the normal at P, in a direction such as PA. Another ray such as OQ will also be refracted away from the normal at Q in a direction such as QB. These two rays form a divergent beam, and an observer stationed in the direction of this beam will deal with it in the manner already described on p. 84, and he would see the object O at O′. The exact position of O′ depends on the particular rays selected as coming from O. A ray such as OR when it emerges into the air will come out very nearly grazing the water-air interface as shown by RC, and it will be easily seen that a divergent beam coming out in this direction when produced backwards would form an image of O very near the surface. By getting down very nearly to the level of the surface of a river, the image of the river bed appears to be almost at the top of the water.

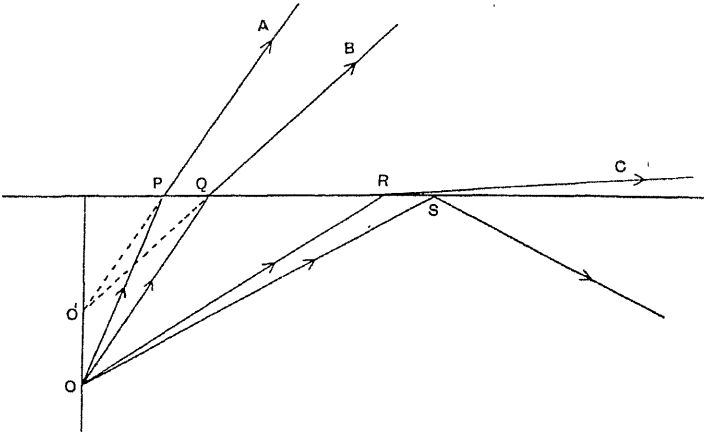


FIG. 50A.

From Fig. 50A it will be seen that a ray a little more inclined than OR would emerge parallel to the interface, and applying Snell's law to this case, we may write

$$\frac{\text{Sin } i}{\text{Sin } r}=\mu,$$

where μ is the refractive index of a beam of light going from air to water. Since in this case the beam is considered as going from water to air, the expression should be written

$$\frac{\text{Sin } r}{\text{Sin } i}=\frac{1}{\mu}.$$

Since sin 90° equals unity, the expression becomes

$$\text{Sin } r=\frac{1}{\mu}.$$

The angle which the ray in question makes with the normal is called the critical angle for the two media. A ray such as OS which is a little more inclined to the surface would not emerge into the air at all, but would be totally reflected back into the water, conforming to Snell's law of reflection. It is to be noticed that all rays from O which strike the interface at S or beyond this point would be completely reflected back into the water. This phenomenon is known as total internal reflection, and occurs whenever light passes from one optical medium to another which is less dense. In the case of water and air, the critical angle can be determined with considerable accuracy, and the value of the refractive index deduced from the relation

$$\text{Sin } r=\frac{1}{\mu}$$

is 1·34.

By completing the diagram in Fig. 50A, it will be seen that the light which emerges into the air is contained within a cone, the semi-angle of which is such that its sine is equal to $\frac{1}{1\cdot34}$, *i.e.*, ·746.

Ordinary crown glass has a greater refractive index than water, and consequently the critical angle will be smaller. Glass prisms are often cut in such a way that light entering one face of the prism is totally reflected at the glass-air interface, the prism thus serving as a perfect reflector.

Spherical Mirrors.—If O is a luminous point on the axis of a curved mirror of which C is the centre of curvature (Fig. 51), the image is found in just the same way as it would be if it were a plane mirror. Select two rays starting from O, including between them an angle C. The ray OA will be reflected along AI and the ray OP will retrace its path because it is incident normally at P. These two rays intersect at I, which is the image of O. From the figure we see that the angles a, b, c bear certain relations to one another, for

$$a = i + b \text{ and } b = i + c.$$

$$\therefore\ a = 2b - c, \text{ or } a + c = 2b.$$

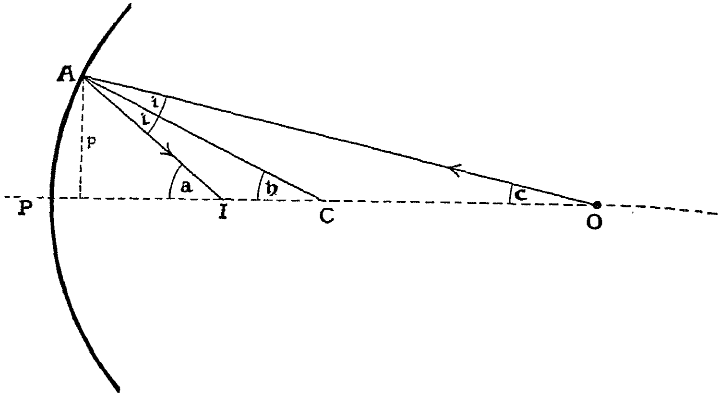


FIG. 51.

We can convert this relation between the angles into one between distances. If OP = u, CP = R, and IP = v, then provided the angles a, b, and c are small, we may take the numerical value of their tangents as being proportional to their circular measure.

Hence
$$\frac{p}{\text{IP}} + \frac{p}{\text{OP}} = \frac{2p}{\text{CP}},$$

or
$$\frac{1}{v} + \frac{1}{u} = \frac{2}{\text{R}}.$$

This relation between the distances of object, real image, and radius of curvature of a converging mirror will be found very useful in checking the accuracy of drawings.

For the beginner the essential thing in optical work is that he should draw to scale all the typical cases of image formation, and check the accuracy of drawing by analytical expressions suitable to particular cases.

Suppose that AB is an object held in front of a concave mirror whose centre of curvature is C (Fig. 52). Provided the aperture of the mirror is not too large, it is found that light parallel to the axis converges after being reflected by the mirror to a point F called the focus.

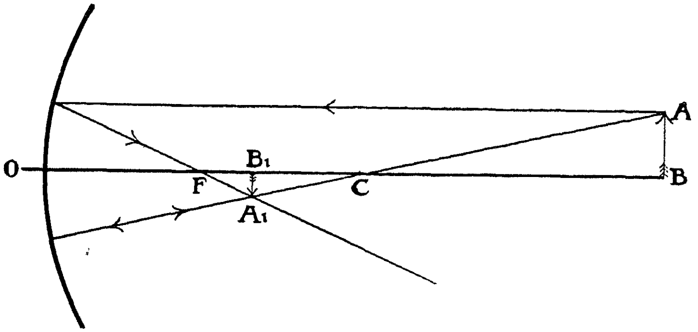


FIG. 52.

F is found to be half-way between O and C, C being the centre of curvature. Therefore the focal length $= \frac{1}{2}$ radius of curvature. In order to get the image of A, at least two rays must be chosen in order to find where they intersect. Any two rays may be selected, but it will be found to simplify the drawing a good deal if two particular ways are chosen :—

1. The ray from A drawn parallel to the axis after reflection goes through F.
2. The ray from A directed through C after reflection goes back upon its own path because it hits the mirror normally. Where these two rays from A intersect is the image of A.

Proceeding in this way for intermediate points between A and B, we get the whole image A_1 B_1.

In the case of a mirror curved the other way, *i.e.*, a convex mirror, if O is a point object, a beam diverging

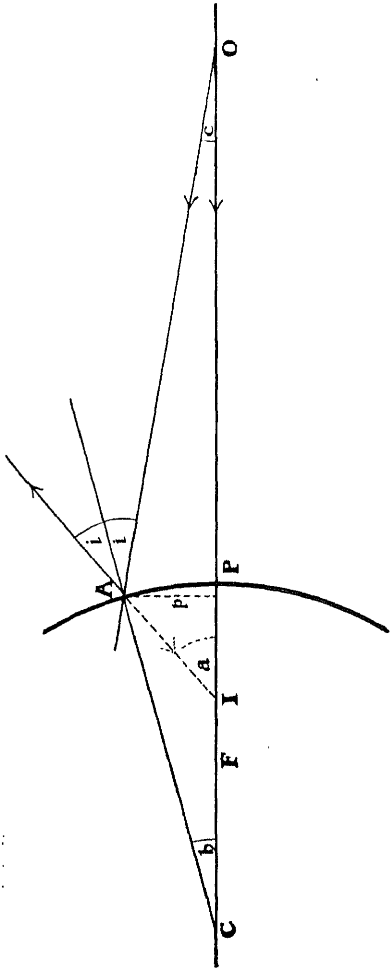


Fig. 53.

from it at an angle *c* meets the mirror at A and P; at A it is reflected away in the way shown in Fig. 53.

At P it is reflected back along its path; these two reflected rays make a more divergent beam than actually started from O and, of course, no real image is formed.

If, however, the reflected rays are traced backwards, they appear to converge to I, hence we say that a virtual image of O is formed at I.

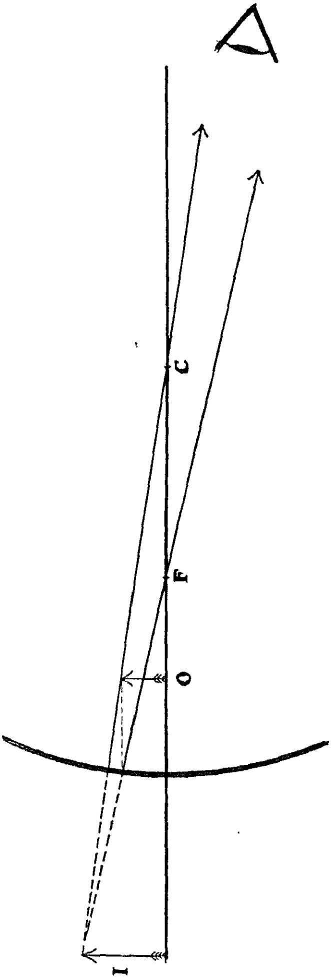


Fig. 54.

Here again we have the angle

$$a = \text{angles } i + b,$$

and $$i = b + c. \quad \therefore \; a = 2b + c, \text{ or } a - c = 2b.$$

Provided the angles *a*, *b*, and *c* are small, the tangents of the angles may, as before, be taken as proportional to the angles themselves.

Hence, if *u* and *v* are the distances of object and image from the mirror, we have

$$\frac{p}{v}-\frac{p}{u}=\frac{2p}{R},$$

where R is the radius of curvature.

$$\therefore \frac{1}{v}-\frac{1}{u}=\frac{2}{R}.$$

Since a convex mirror tends always to disperse or diverge light falling upon it, no real images can be formed with it, wherever the object O may be placed. It has already been shown that the function of a concave mirror is to converge light, so it tends to form real images of objects; but if the object is held too near such a mirror, the divergence of the rays from the object may be too great for the converging power of the mirror to overcome it. These two tendencies will, as a matter of fact, just balance if the object is put at the focus of the mirror; and in such a case, since the divergence is just, and only just, overcome by the mirror, the light will have neither tendency, *i.e.*, it will be parallel.

If the object O is brought inside the focus F and the same general lines of geometrical construction are attempted, it will be seen that the light diverges after it leaves the mirror, as in Fig. 54.

If an eye is placed so as to receive this divergent beam, it appears to see an image of O at I. This image is a virtual image, and the construction shows that it is an enlargement of O; the exact position of this image may be found by means of a formula for concave mirrors derived as follows :—

Let C be the centre of curvature of the mirror (Fig. 55), O an object placed within the focus. Any ray from O except the one incident at the pole of the mirror will be reflected outwards, as in Fig. 55. The one so drawn makes with the one reflected through C a divergent beam; if this beam is received by the eye, it sees an image at I. Then, as before, the angles

$$a=i+c,\quad i=b+c.\quad \therefore\ a=b+2c,$$

or

$$\frac{p}{u}=\frac{p}{v}+\frac{2p}{\mathrm{R}},\quad \textit{i.e.},\quad -\frac{1}{v}+\frac{1}{u}=\frac{2}{\mathrm{R}}.$$

If we compare this expression with the one derived for the case of a concave mirror producing a real image of an object, we see that a minus sign comes in. This is very important when using the expression to check the results of graphical work.

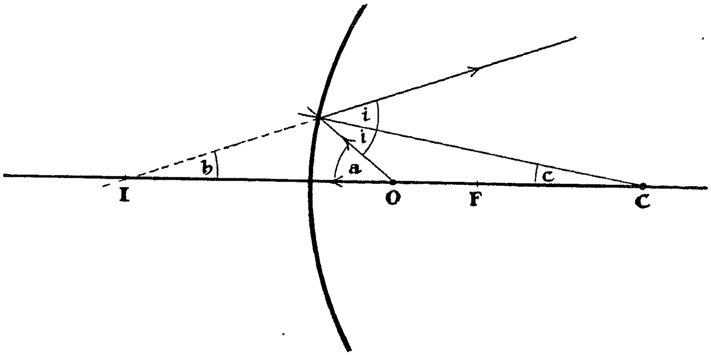


FIG. 55.

As shown in Fig. 49, a ray of light is refracted as it goes from air to glass, and if the glass is a plane parallel slab, the light as it comes out into the air is refracted again, but in the opposite sense, so that it emerges in the same direction as it went in.

If, however, the two surfaces of the glass are inclined to one another, we have what is called a prism, and the rays do not emerge parallel to their original direction; they suffer deviation, as in Fig. 56. An eye placed to intercept this divergent beam deals with it in a way very similar to what happens to a divergent beam from a mirror (Fig. 50); the observer sees an image at I where the rays, produced backwards, appear to meet; I is the virtual image of O. The angle δ which the divergent ray makes with the initial direction of the beam is called the deviation. It is found experimentally that if the prism is rotated so that the incident beam falls on it at a different angle, the position of I changes; it may move downwards towards O, or it may move further away from O. Obviously, the nearer I moves towards O, the less the rays are deviated by the prism;

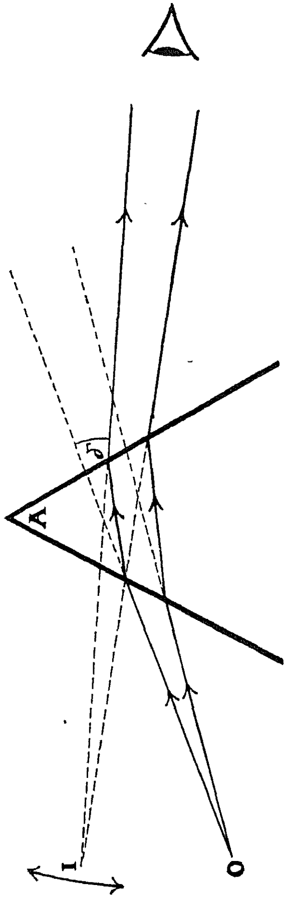


Fig. 56.

for any prism there is one definite position in which the minimum deviation occurs, so that whether it is rotated clockwise or anti-clockwise as we look through the prism, the image becomes more deviated.

This particular position is called the position of minimum deviation, and in this case the rays go through the prism symmetrically, *i.e.*, the angle at which they come out is equal to the angle at which they go in.

Let OP (Fig. 57) be the incident ray, i the angle of incidence; this ray is refracted along PR and at R once again making an angle i with the normal. Then by geometry we have

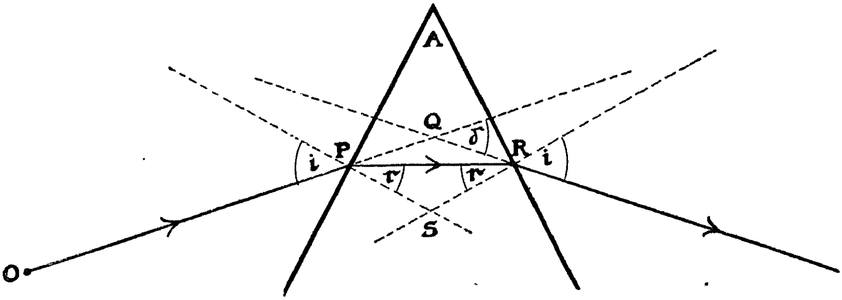


Fig. 57.

$$\delta = 2(i - r),\ A = 2r,\ \text{and}\ \frac{\operatorname{Sin} i}{\operatorname{Sin} r} = \mu.$$

Substituting for i and r, we have

$$r = \frac{A}{2}\ \text{and}\ i = \frac{\delta + A}{2}.$$

$$\therefore\ \mu = \frac{\operatorname{Sin} \frac{\delta + A}{2}}{\operatorname{Sin} \frac{A}{2}}$$

Hence measurements of the angle of a prism and of the minimum deviation produced by it give us the refractive index of the material of the prism.

If we take a prism of very small angle (Fig. 58), the expression for the deviation becomes a very simple one. If we consider that the incident light makes a very small angle with the normal, then $\mu = \frac{\text{Sin } i}{\text{Sin } r}$ may be written $\mu = \frac{i}{r}$; and since the deviation $\delta = 2(i - r)$ and $A = 2r$, we have by substitution

$$\delta = (\mu - 1)A.$$

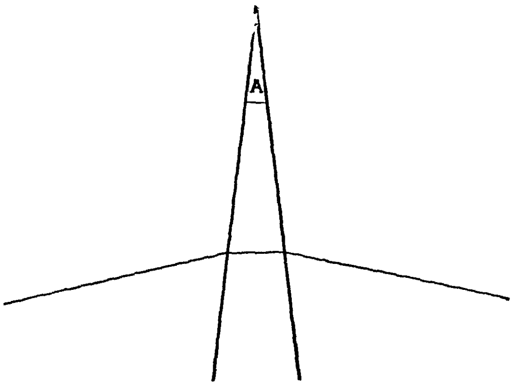


FIG. 58.

Hence the deviation produced by a small-angled prism is proportional to this angle.

Imagine three prisms made up of different angles; truncate the one of smallest angle and put the one of middle angle on top of it; truncate this one and put the one of biggest angle on it, and we shall have an arrangement as shown in Fig. 59.

Light from a luminous point O incident upon this arrangement will be brought to a focus at O′, for the truncated prisms can deviate rays of differing divergence to an increasing extent as we go upwards to the apex of the final prism.

If these truncated prisms gradually merge one into the other, we have what is known as a lens. These are of two main kinds, convex or converging lenses and concave or diverging lenses.

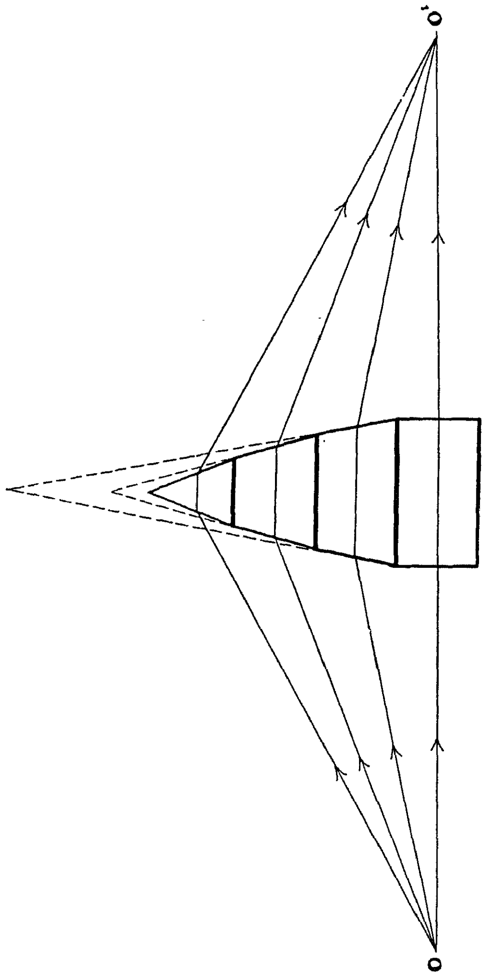


Fig. 59.

The most usual form of converging lens is the double convex lens (Fig. 60), consisting of two surfaces of equal but opposite curvature. If a parallel beam of light falls on such a lens C, the beam is made to converge and is brought to a focus at F; the distance of this point from the centre of the lens is called the focal length *f*. The more converging the lens, the shorter this length. In fact, the converging power of a lens is measured by $\frac{1}{f}$. If *f* is measured in metres, this reciprocal is termed the dioptre of the lens. Prescriptions for spectacles are made out in dioptres.

Light from an object AB (Fig. 61), suitably placed in front of a convex lens, whose centre is C, may be converged to form an image, and the real image DE is seen to be inverted. The simplest rules for determining the position of the image graphically are as follows :—

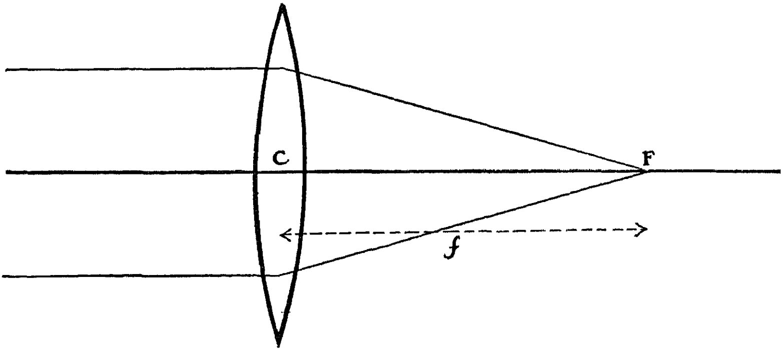


FIG. 60.

From any point, say B, of the object, choose two rays—

1. The ray from B parallel to the axis of the lens; this will, as we have just seen, go through the focus F of the lens.
2. The ray directed from B towards the centre C of the lens; this will go through the lens undeviated, because the two opposite sides of the lens constitute a parallel slab, and Fig. 49 shows that such a slab does not alter the direction of transmitted light.

Hence the divergent beam from B, after going through the lens, becomes a convergent beam which in fact produces an image of B at E. If the same construction

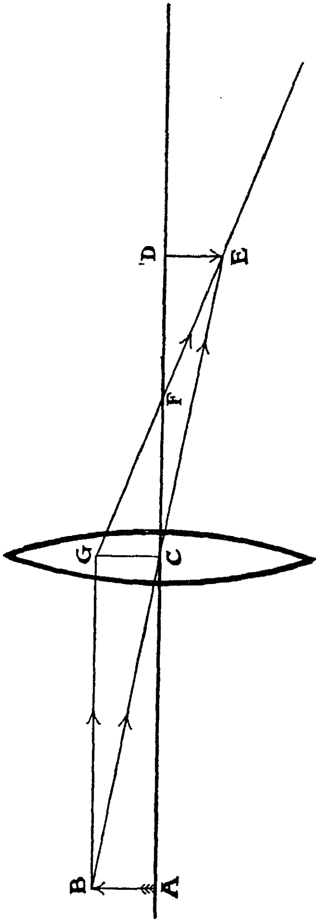


Fig. 61.

be carried out for intermediate points between A and B, the image DE is obtained.

A simple relation can be found between u, v, and f, where u represents the distance of the object and v the distance of the image from the lens and f the focal length.

F SCIENC

The triangles DEF, GCF are similar.

$$\therefore \frac{DE}{GC} = \frac{DF}{CF}.$$

Also the triangles DCE, ACB are similar.

$$\therefore \frac{DE}{AB} = \frac{DC}{AC}, \text{ and by construction } AB = GC.$$

$$\therefore \frac{DF}{CF} = \frac{DC}{AC}.$$

Hence $$\frac{v-f}{f} = \frac{v}{u}, \text{ i.e., } uv - uf = fv.$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

It will be noticed that this is a similar relation to that obtained when a real image is produced with a concave mirror.

MAGNIFICATION.—The ratio of the size of an image to the size of the object is called the magnification. From inspection of Fig. 61, we see that

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{Distance of image from lens}}{\text{Distance of object from lens}},$$

a simple ratio giving the magnification.

If we now consider what would happen if a lens were constructed out of truncated prisms placed in the way shown in Fig. 62, it is seen that light would tend to be diverged instead of converged.

If an eye be placed to the right-hand side so as to collect this divergent beam, it would appear to see O at I, and I is called the virtual image of O.

A lens constructed on these lines is called a concave lens, its function being to diverge light; parallel light is diverged by such a lens, but, looking at a very distant object through a concave lens, a diminished image of the object appears at a distance called the focal length of the lens, so that although light is not really brought to a focus at all by such a lens, the term

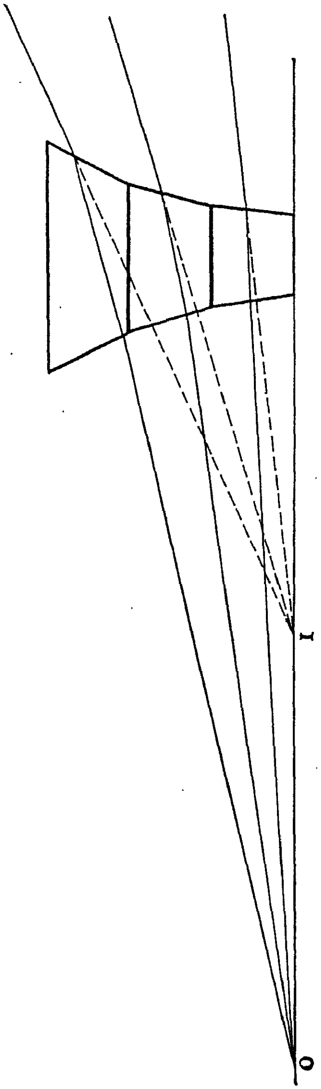


FIG. 62.

focal length is used in the same general sense as for a convergent lens.

Let AB be an object held at a distance from a concave lens (Fig. 63). The ray from B parallel to the axis i diverged by the lens, as shown in Fig. 62; the ray directe to the centre C is undeviated; these two divergent ray do not form a real image, but they appear to diverg from E where ED is the virtual image of AB; if v is th

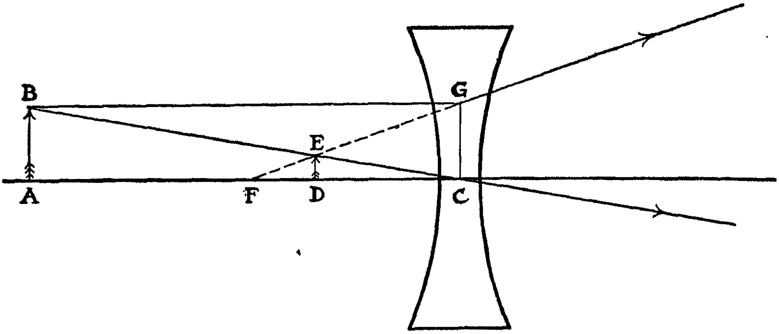


FIG. 63.

distance DC, then by similar triangles we have as before :—

$$\frac{FD}{FC}=\frac{DC}{AC}$$

$$\therefore \frac{f-v}{f}=\frac{v}{u}, \text{ or } uf-uv=vf.$$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}.$$

This is a similar expression to that found when a virtual image is produced with a convex mirror.

If an object is brought within the focal length of a converging lens and the same two rays are taken as before, the ray parallel to the axis will then go through F and the ray directed to C will pass undeviated through the lens. These form a divergent beam; hence no real image can be formed. But if an eye is placed to collect this beam, it sees a virtual image at A′B′, and from the construction of Fig. 64 it is clear that it must be a magnified image of AB. This is how a simple magnifying glass acts.

When a virtual image is formed with a concave mirror, the position of the image can of course always

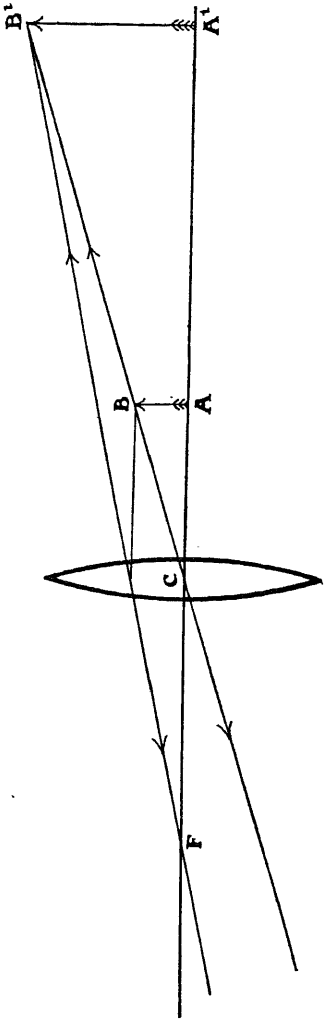


Fig. 64.

be found graphically, and to check the accuracy the usual formula with a negative sign for v is used. In exactly the same way when dealing with the formation of a virtual image by a convex lens, careful drawing can be checked by using the formula for a convex lens, but with a negative sign for v. For example, we may take an object held 2 in. away from a lens of focal length $2\frac{1}{2}$ in.

The construction shows that the virtual image is found at a distance of about 10 in. from the lens.

Using the formula

$$-\frac{1}{v}+\frac{1}{u}=\frac{1}{f},$$

and substituting, we have

$$-\frac{1}{v}+\frac{1}{2}=\frac{1}{2\cdot 5}.$$

$$\therefore\ \frac{1}{v}=\frac{1}{2}-\frac{1}{2\cdot 5}.$$

$$\therefore\ v=10\text{ in.}$$

Some physicists prefer to deal with these two cases of virtual image formation which need a change in sign for v in a different way. All of the formulæ for mirrors and lenses are derived with the following convention as to signs: distances measured on the same side of the mirror or lens as the object are considered positive; those on the opposite side negative. While there is much to be said for this method, it may be easier for the beginner to retain the essential ideas of convergence and divergence, and the formulæ which have been derived with no conventions at all, and simply remember that when dealing with virtual images produced by converging mirrors or lenses, a change in sign of v must be made. There is then no need for looking upon the focal length of a converging lens as a negative quantity.

A convex lens is a device for converging light, and its power of doing so is briefly expressed as $\frac{1}{f_1}$, where f_1 is its focal length; if we place another lens whose converging power is $\frac{1}{f_2}$ close to the first one, then the resulting convergence is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}.$$

If, however, we combine with the first one a diverging lens whose power is given by $-\frac{1}{f_2}$, then the resulting convergence is

$$\frac{1}{F}=\frac{1}{f_1}-\frac{1}{f_2}.$$

Whether the double lens will converge or diverge light depends, of course, upon whether $\frac{1}{f_1}$ is greater or less than $\frac{1}{f_2}$.

In the case of curved mirrors, the focal length is one-half of the radius of curvature. For lenses no such simple relation holds. It can be proved that

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right),$$

where μ is the refractive index of the glass and R_1, R_2 the radii of curvature of its two curved surfaces. In the special case where $\mu=1{\cdot}5$ and $R_1=R_2$, the focal length is just equal to the radius of curvature.

THE EYE.—Normal sight is made possible by one of the most perfect structures exhibited by living things. The eye, a diagram of which is given in Fig. 65, is made up of a thick covering called the sclerotic coat, which is lined, except for the actual aperture of the eye, with the retina, which plays the part of a screen upon which the optical images are formed; these images give rise, through a very elaborate mechanism, to sensations which are conveyed to the brain by means of the optic nerve. The cornea is the thin outside lens separated from the lens proper by a fluid called the aqueous humour. The lens is suspended in position by filaments attached to the ciliary muscles; the inner chamber of the eye is filled with the vitreous humour. The adjustments necessary for the clear seeing of objects on the horizon and at the next instant the type of a book held a few inches away from the eye are so perfect that one is not even conscious that anything is being adjusted.

This process is called accommodation, and it consists in altering the curvature of the lens to suit the divergence of the rays entering the eye. When looking at a very distant object, the eye has to cope with rays having hardly any divergence at all, in fact practically parallel light, so the lens needs its minimum curvature and it is then relaxed. When reading a book there is a demand for more curvature of the lens, and if the book is brought too near to the eyes, a distinct sense of effort is felt in order to see the very near object distinctly.

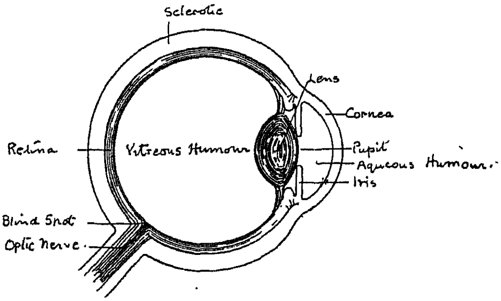


FIG. 65.

One can soon find for oneself that there is a best distance for viewing an object well, and this is referred to as the minimum distance of distinct vision, and for most people it is about 9 in. from the eye.

That images are really formed on the retina can very easily be shown by taking a large eye, say the eye of an ox, cutting a small window in the retina in line with the pupil and cementing a thin glass window over the aperture to retain the vitreous humour; a thin layer of paraffin wax over the window allows one to see the image of a distant object quite well.

DEFECTS OF VISION.—There are four main defects of vision which can be compensated by comparatively simple means. They are :—

Myopia, or short sight.
Hypermetropia, or long sight.
Presbyopia.
Astigmatism.

People suffering from myopia cannot see distant objects distinctly; very often they cannot see things clearly more than a foot away; when fully relaxed, the lens is still too convergent for distant objects; images are therefore formed in front of the retina with consequent blurring of vision. The defect is remedied by giving the fully relaxed lens just the amount of divergence it needs to focus a distant object, so the spectacles used are concave lenses.

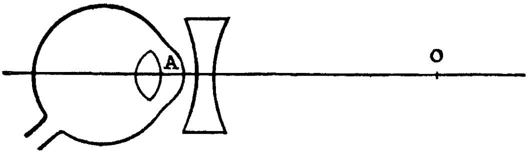


FIG. 66.

If AO (Fig. 66) is the greatest distance off that an object can be and still be seen clearly by a myopic person, O is called the far point, and all points from O to infinity, or say the horizon, have to be corrected. If a concave lens of focal length AO be put in front of the eye, then a distant object will give a virtual image at O which the eye could cope with. As the object is brought nearer and nearer to the eye, clear vision is brought about by the natural process of accommodation.

People suffering from hypermetropia can see distant objects clearly enough, but when anything is brought nearer than about a yard from the eye, it cannot be seen clearly because the distance between the lens and the retina is unusually short. The remedy here is to help the eye with just the amount of converging power that it lacks. Clearly the greatest amount will be needed when the long-sighted person wants to see an object at the normal minimum distance of distant vision. Imagine a case so that the long-sighted person has what is called a near point N a yard away, *i.e.*, he cannot see clearly any object nearer to him than this. He wants to see objects between N and **D** (Fig. 67), where **D** is 9 in. from his eye.

If he is given a converging lens of such power that a virtual image of an object at **D** can be formed at N, he will then be able to see the object clearly, and an object anywhere between **D** and N will be dealt with by the natural process of accommodation.

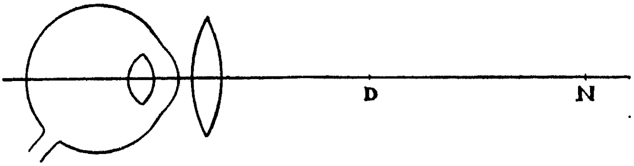


FIG. 67.

The focal length of the lens required can be found graphically, as in the example of Fig. 64, or by means of the equation

$$-\frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

We have

$$-\frac{1}{36}+\frac{1}{9}=\frac{1}{f}.$$

$$\therefore f=12 \text{ in.}$$

Presbyopia is essentially a defect of vision that gets more pronounced with age. A presbyopic eye has increasing difficulty in seeing clearly near objects, due to a lack of converging power of the lens. This is remedied by the use of convex lenses, as in the case of hypermetropia.

Astigmatism is a defect of vision due to the lens having different curvatures in different directions. Suppose that a vertical section through the lens showed a greater curvature than that given by a horizontal section. Then rays coming from an object held in front of such a lens will not be brought to a proper focus owing to the asymmetry of curvature. An extreme case of such difference of curvature is met with in cylindrical lenses; here a definite curvature in one plane is associated with no curvature at all in a direction perpendicular to it. The way of correcting astigmatism is to provide a lens with just the degree of curvature necessary to make good the deficiency found in the lens of the eye.

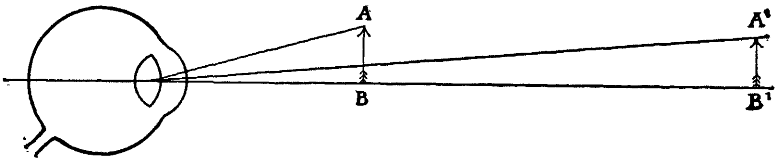


Fig. 68.

Aids to Normal Vision.—The simplest aid to normal vision is the ordinary magnifying glass. The clearness with which we can ordinarily see a thing depends upon the angle which objects subtend at the eye. For example, an object at AB (Fig. 68) is seen more clearly than at A′B′, because it subtends a bigger angle at the eye; this is easily tested by taking a sheet of small print well illuminated and holding it, say, a couple of feet from the eye; in order to distinguish the print most easily, we bring it to the minimum distance of distant vision, *i.e.*, about 9 in. from the eye. The angle of vision would, of course, be more favourable still if we brought it nearer, but the accommodating power of the eye would become strained in order to deal with the great divergence of the rays entering the eye. If, however, with the object quite near the eye we interpose between them a convex lens, we can arrange that its virtual and therefore magnified image is formed at AB; in this way the magnifying power of a simple lens is explained.

If a convex lens of focal length 3 in. is held close to the eye and the object is held $2\frac{1}{4}$ in. from the lens, a virtual image is formed 9 in. away from the lens. This can be proved by drawing, as in Fig. 64, or from the expression

$$-\frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

If for v the distance of the image from the lens we use the letter D to denote the minimum distance of distinct vision, then we may write

$$-\frac{1}{D}+\frac{1}{u}=\frac{1}{f},\ \textit{i.e.,}\ \frac{1}{u}=\frac{1}{f}+\frac{1}{D},$$

or

$$u=\frac{fD}{f+D}.$$

Magnification is defined as the ratio of the size of the object to the size of the image, and, as shown in Fig. 61, this is the same as the ratio $\frac{v}{u}$.

Hence the magnification produced by a single lens used as a magnifier is in this case given by $\frac{D}{u}$.

Substituting the value of u, we have

$$\text{Magnification}=\frac{D}{\frac{fD}{f+D}}=\frac{f+D}{f}=1+\frac{D}{f}.$$

With the values given in the example, it is seen that the magnification will be $1+\frac{9}{3}=4$, a result which agrees with that given by the ratio

$$\frac{D}{u}=\frac{9}{2\frac{1}{4}},\ \textit{i.e.,}\ 4.$$

A simple microscope consists of two lenses; the one near the object is called the objective and the one near the eye the eye-piece. Suppose that it is desired to look at a very small object AB (Fig. 69); if we place

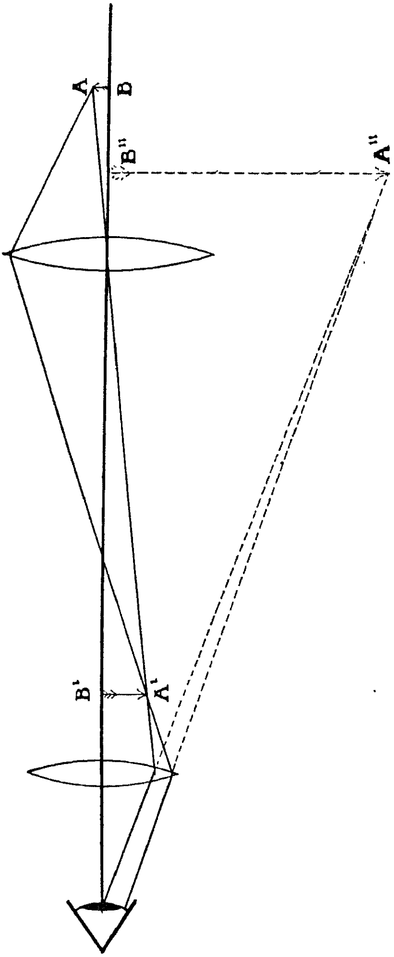


FIG. 69.

it just outside the focus of a very convergent objective, *i.e.*, one of very short focal length, a magnified image appears at A′B′; if the eye-piece is now arranged so that this real image A′B′ is within the focus of the eye-piece, a virtual and enlarged image $A''B''$ appears. It is usual to arrange that this image is formed at the minimum distance of distinct vision for reasons just stated, and it will be noticed from Fig. 69 that it is an inverted image of the object; but people who use microscopes a good deal soon get used to this. There are many very important developments embodied in these simple principles which make the modern microscope the excellent instrument it is, but these cannot be entered into here.

A telescope is designed to help the vision in looking at distant objects. Here again in the simplest form of instrument there is an objective and an eye-piece, but in this case the objective is one of long focal length.

Rays from a very distant object form an image CD (Fig. 70) only just a little beyond the focus of the objective; the eye-piece is, as in the microscope, adjusted so that this image is just within its focal length, and therefore gives a magnified virtual image HK. The magnifying power of a telescope is given by the ratio of the size of the image to that of the object. This ratio may be expressed as the ratio of the angles subtended by image and object. The telescope itself is short compared with the distance of the object, so the half angle subtended at the eye by the object is α. In the diagram only one-half of the field of view is drawn. The eye is held close to the eye-piece, so that the half angle subtended by the image is β.

Hence magnifying power

$$=\frac{\beta}{\alpha}=\frac{\frac{CD}{ED}}{\frac{CD}{FD}}=\frac{FD}{ED}.$$

FD is very nearly the focal length of the objective and ED is practically the focal length of the eye-piece, so we may say that the magnifying power of telescope

$$=\frac{\text{Focal length of objective}}{\text{Focal length of eye-piece}}.$$

The image formed by a simple telescope is inverted which, though not a serious matter in astronomical work, would obviously be inconvenient for observation

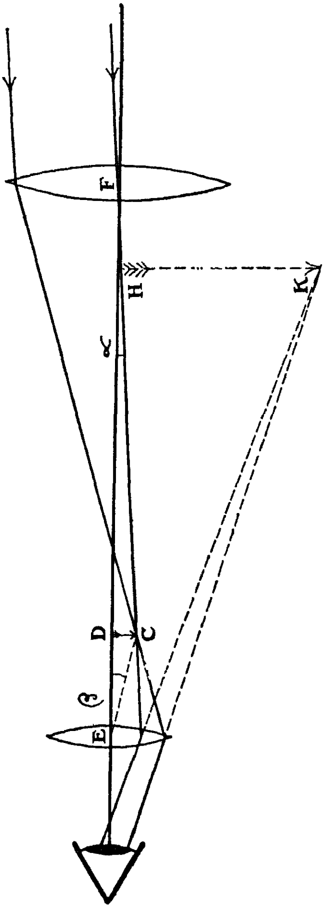


Fig. 70.

over land or sea. Terrestrial telescopes are therefore provided with extra optical parts which ensure that the image is the right way up; this is usually done by having two convex lenses of equal focal length set up between the objective and eye-piece in such a way that the distance between them is equal to twice their focal length.

SPECTRA

The Solar Spectrum.—Newton showed in 1672

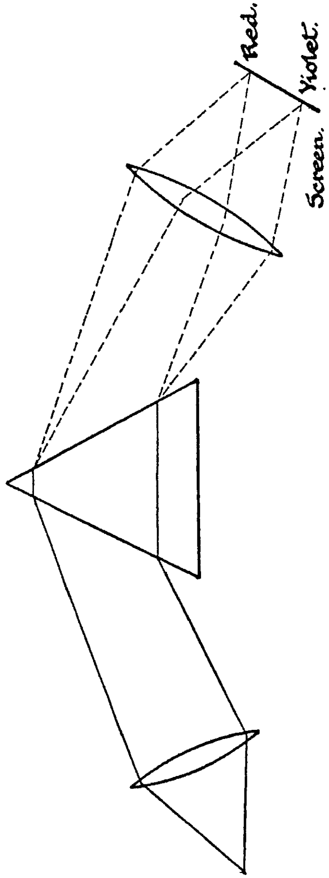


Fig. 71.

that a beam of white light can be split up by means of a prism into a number of constituent colours. He arranged that a beam of sunlight fell upon a glass prism in such a way that the beam as a whole was deviated out of its course. Newton observed that the emergent beam was no longer white but coloured, and by collecting the emergent beam with another lens, the different components of the beam were brought to a number of foci, as shown in Fig. 71. Newton called this the spectrum of sunlight, and the term is still used to denote the various constituents into which any source of light may be analysed.

Sunlight may be resolved into what are known as the spectral colours; they are red, orange, yellow, green, blue, indigo, and violet. Red light is deviated by the prism least; the intermediate colours come in between the red and the violet in the order in which they are named.

Since Newton's discovery a great deal more has been found out about the composition of light, and one of the most important things is that the different colours have different wave-lengths. Perhaps our first impressions of wave-length are got from watching waves on the sea. The distance from the crest of one wave to the crest of the next is called the wave-length, and in water we know that this may vary from quite tiny ripples to the big Atlantic rollers which may be separated from one another as much as 50 ft.

It is possible to measure the wave-lengths of the various coloured constituents of white light, and it is usual to express these wave-lengths in what are called Ångstrom units and written Å.U. The wave-lengths are very small indeed, but it will be seen in a later section that there are rays which have wave-lengths far shorter than light. Red and violet rays (Fig. 72) set the limits at either end of the visible spectrum; the actual limits of vision are generally taken for anyone of normal sense of colour to extend from a wave-length of 7,600 Å.U. (red) to 3,800 Å.U. (violet). Some people have a little bigger range of vision than this, but many have not quite so big a range. If we look at a spectrum projected on to a screen, we notice that there are parts, for instance, where it appears yellow, and this extends for some distance until we begin to see a change, say, to green; this in turn extends some distance until we detect a bluish tinge appearing, and so on. Although we may look an appreciable distance across the spectrum without the eye detecting any appreciable change in the colour, yet any appreciable shift across the spectrum means an actual change in wave-length ; so that a whole group of wave-lengths are included in what we call the yellow

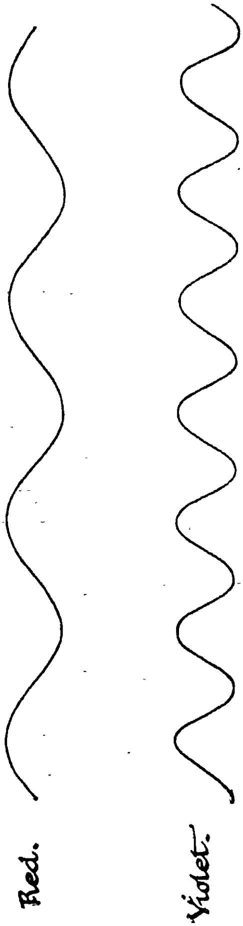


FIG. 72.

part of the spectrum and another group in the green, and so on.

The most usual way of examining the spectrum of a body is to follow very closely the lines laid down for the production of the solar spectrum by Newton, only visual observation is often dispensed with in favour of a photograph of the spectrum. The instrument for observing spectra is called a spectroscope, a diagram of which is seen in Fig. 73.

The light from the source S, whose spectrum is to be examined, passes through a narrow vertical slit N placed at the focus of a lens L_1.

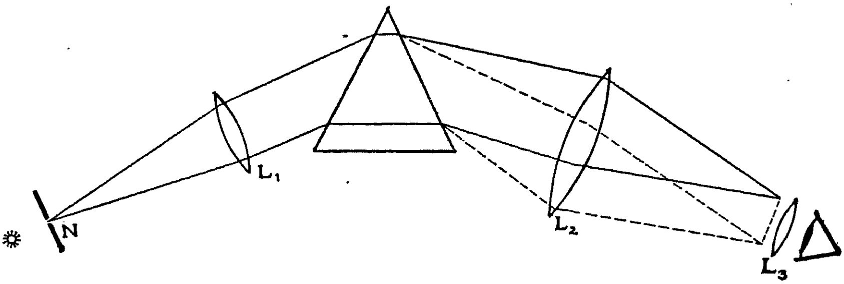


FIG. 73.

This part of the instrument is called the collimator. From L_1 the light falls on a prism and the components of light are dispersed by it to different extents, so that they are brought to different foci by the lens L_2. The spectrum which they then form is viewed by the observer through the eye-piece L_3. If a photograph of the spectrum is wanted, the eye-piece is taken out and a photographic plate is put so that the spectrum comes sharply in focus on it.

Suppose that S gave out light of only one wave-length; these waves would enter the slit N, be collected by L_1, and in turn by L_2, after being deviated by the prism, be brought to a focus by L_2 forming in fact an image of the slit N; if N were made a very fine slit, all we should see on looking through the eye-piece L_3

would be a very thin line. The larger the number of different wave-lengths given out by S, the larger the number of lines that will be seen in the field of view of the eye-piece. These lines are often called spectral lines and, as explained, they are simply images of the slit of the collimator. By widening the slit the spectral lines are broadened; by narrowing it the lines are got very fine.

Emission Spectra.—The spectra which are given out by bodies vary very largely with experimental conditions. We can, however, say that in nearly all cases when a body is giving out light (not reflecting or scattering it from some other source) it is also giving out heat, but that in ordinary spectroscopic observations this is lost sight of, because the eye does not detect radiant heat easily unless it is intense.

Now one kind of spectrum is produced by the simple process of heating a body; when a poker is put into a fire, it soon becomes a dark red colour, then bright red, then yellowish, and finally, with a very hot fire, the poker looks almost white; this is known as "white hot."

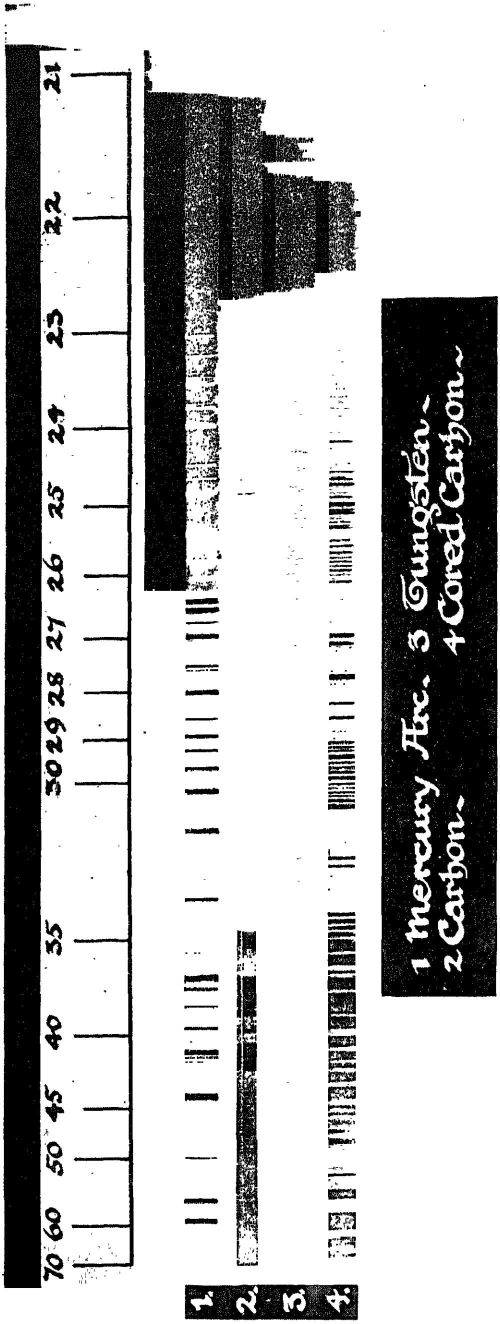
If the poker be examined spectroscopically at these various stages, it will be found that initially the spectrum consists of red light, but finally all the colours of the spectrum will be represented in it; very similar indeed to the solar spectrum. If these observations were made with a poker made of either copper or silver or any other metal of high enough melting point, exactly the same sort of thing would be noticed—the spectrum actually seen would depend only on the temperature and not on the particular metal chosen. Such a spectrum is known as a continuous spectrum, and this is often referred to as temperature radiation.

By changing the experimental conditions, these three metals may, however, be shown to be capable of emitting spectra quite different from one another. One way of showing this is to form an electric arc of the metal; this is done by fitting two pieces of the metal, called poles, into clamps in such a way that, when the two pieces of metal touch one another, they complete an electrical circuit; at the moment of contact an electrical current passes from one pole to the other and the pole pieces are then withdrawn from one another; but the current goes on flowing between them as some of the metal is volatilised. Bringing the two pole pieces together, and separating them is called striking, and the arc that is formed will go on until the pole pieces are volatilised to such an extent that the gap between the poles becomes too big for the current to be conveyed across; the arc will then suddenly go out unless the pole pieces are brought nearer together. This is done by some automatic mechanism in the large carbon arcs which are used for street lighting, cinema projectors, searchlights, etc.

In marked contrast to the spectra of merely hot bodies, the arc spectra differ enormously from one another. In Fig. 74 the arc spectra of a number of different substances are grouped together; those chosen are often used for medical purposes. Two of the most noticeable features of these arc spectra are that they appear to consist very largely of lines and bands and, in order to insist upon the distinction between them and the continuous spectra already discussed, these spectra are often called line and band spectra; band spectra appear to be disjointed portions of continuous spectra except that they have certain added characteristics.

If a series of sparks from an induction coil is sent from one pole piece to another of the same metal, which is only a few millimetres away, and the light from the sparks is then examined with a spectroscope, it is found to be quite different from the arc spectrum for that particular metal. The spectra so formed are called spark spectra, and generally consist of series of spectral lines; they are of great scientific interest, but they are not put to much medical use.

In the section devoted to photometry, it was stated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4.
1 Mercury Arc. 3 Tungsten.
2 Carbon. 4 Cored Carbon.

FIG. 74.

that many substances other than those termed opaque bodies have the power of absorbing light to some extent. The best way of finding out how much the different components of light are absorbed by a substance is to combine the methods of photometry and spectroscopy.

For example, when a solution of copper sulphate is held up to the light, the transmitted light appears to the eye to be blue. We might from this conclude that all other colours of the spectrum are absorbed by the solution, letting through only blue. If, however, the solution is placed in front of the slit of a spectroscope, so as to intercept the light from an incandescent source and the transmitted light viewed through the eye-piece of the instrument, it is found that this is not the case; many of the other colours are transmitted, though the red and orange are very largely absorbed by the solution. The spectrum so obtained is called the absorption spectrum of the substance, and these absorption spectra are made considerable use of in medicine. One of the most important instances is that of blood. If the end of the finger is pricked with a clean sterile needle and a few drops of blood are shed into a solution of sodium citrate, the blood corpuscles retain their globular form, and when the solution is examined in the way just described, it is seen that parts of the visible spectrum are missing, *i.e.*, they have been absorbed by the blood. Such an absorption spectrum is illustrated in Fig. 75

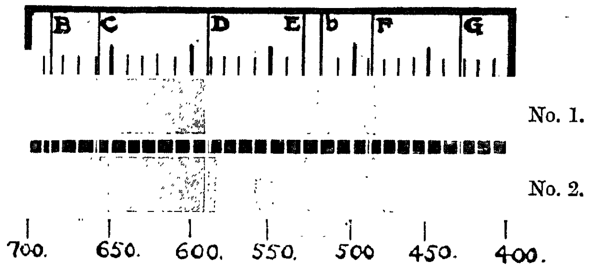

FIG. 75.

(No. 1). The substance in the blood causing this absorption is known to be oxy-hæmoglobin; in cases of carbon monoxide poisoning, it is known that this substance is changed into carboxy-hæmoglobin, which has a different absorption spectrum from that of ordinary oxy-hæmoglobin. This may be seen from No. 2 in the same illustration. There are many other instances in which absorption spectra may be used to identify changes in the fluid constituents of the body.

Reference to the arc spectra of Fig. 74 shows that there are numbers of spectral lines extending far beyond the wave-length 3,800 Å.U., the particular wave-length generally taken to denote the limit of vision for violet light. If an observer had brought his eye to the eyepiece of the spectroscope, he would have seen nothing beyond this wave-length; but when a photographic plate is substituted for the eye, a record of many other spectral lines is obtained, many of which are of much shorter wave-length than the extreme violet of the visible spectrum. Such radiation is called ultra-violet radiation, and although these rays do not act upon the retina of the eye causing the sensation of vision, their existence is not open to doubt when they can be so easily registered on a photographic plate. These rays, which are sometimes called actinic rays, have been found to be very active chemically; they also have very remarkable effects upon living things, many of which are not yet at all perfectly understood. The bulk of the ultra-violet rays from any of the arc lamps which are largely used in medical work penetrate less than a millimetre of one's skin; yet in spite of this, it has been shown indisputably that rickets can be cured by exposing the child's body to these rays; how this is brought about is not yet certain, but it appears to be due to a chemical substance formed by the action of the rays upon the skin in some way and its subsequent liberation into the circulation.

Photographic plates are generally much more sensitive to the violet end of the spectrum than to the red end;

in fact, they are often quite insensitive to red light, and this allows one to develop plates in a pure red light. Plates can, however, be made which are sensitive to red rays, and when the arc spectra mentioned before are photographed on these plates, the photographic record is seen to extend to wave-lengths longer than 7,600 Å.U., the limit of vision for red light. Here again the inference is drawn that the arc spectra give out rays, invisible to the eye, but of longer wave-length than can be detected by the eye. These rays are called infra-red rays; they are not generally investigated by means of photography, for other much more delicate and convenient methods have been devised. The luminous spectrum, therefore, is said to extend between the limits fixed by vision; but sources of light are only too often at the same time sources of infra-red and of ultra-violet radiation.

ACHROMATISM.—The spread of colours by a prism is called dispersion, and is of great importance in the design of optical instruments. In view of this it is necessary to specify the refractive index of optical media in more detail than we have so far done.

For instance, if a beam of red light is allowed to pass through a prism of ordinary crown glass, we find by the method outlined on p. 95 that the refractive index for a beam of red light in going from air to crown glass is 1·511; and on doing a similar experiment with violet light, a refractive index of 1·533 is given; a difference of $1\frac{1}{2}$ per cent. Similarly, with a prism of flint glass, we find for red rays $\mu = 1{\cdot}695$, and for violet rays $\mu = 1{\cdot}743$, a difference of nearly 3 per cent. It is seen then that not only has flint glass a bigger refractive index than crown glass for light as a whole, but that it also has a greater dispersive power.

An ordinary convex lens will have a shorter focal length for violet light than for red light, so that a parallel beam of white light cannot be brought to a single focus, and if such a lens is used in a camera, the picture on the viewing screen, although in focus for one sort of light, is out of focus for the others. This defect of lenses is called achromatism; it can be corrected by making use of two kinds of glass for the particular lens that is required. Suppose, for instance, that a convergent lens of crown glass is combined with a divergent lens of flint glass. A parallel beam of white light entering from the left of Fig. 76 will have its violet components more deviated than the red components by the convex lens; but when this separated beam falls on the divergent lens of flint glass, this separation tends to be corrected, and by suitably choosing the focal length, so that it is not actually as much divergent as the crown glass lens is convergent, the combination of the two lenses may still act as a convergent lens; but there is no spreading out of the constituent colours as there would be if either lens were used separately. This method is the one generally used for getting rid of chromatic aberration, and the combination of lenses is called an achromatic lens.

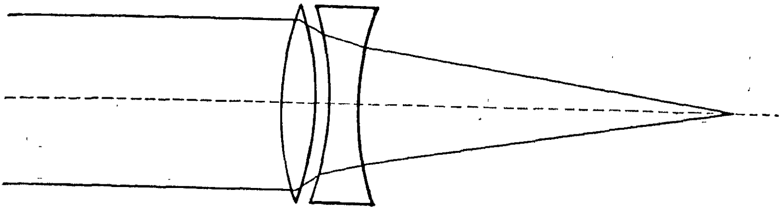

Fig. 76.

Advantage is taken of the different dispersive powers of crown glass and fluid glass in the construction of what is called a direct-vision spectroscope. Suppose that we have an ordinary crown glass prism and that we combine with it one of flint glass of rather smaller angle inverted as shown in Fig. 77. A beam of white light will be deviated by the first prism, and there will be dispersion of the different colours; but the tendency of the second prism will be to deviate the rays in the opposite direction, and it is a matter of adjustment of the two angles of the prisms to arrange that, on the whole, yellow light is quite undeviated in going through the pair of prisms. The condition will be seen, by reference to the case on p. 96, to be approximately that

$$(\mu_c - 1)A_C = (\mu_f - 1)A_F,$$

where μ_c and μ_f are the refractive indices for yellow light in going through the crown and flint glass prisms, and $A_C A_F$ are the angles of the two prisms.

Under these conditions the red and violet components of the beam of white light will be deviated slightly to either side of the yellow, so an instrument is evolved

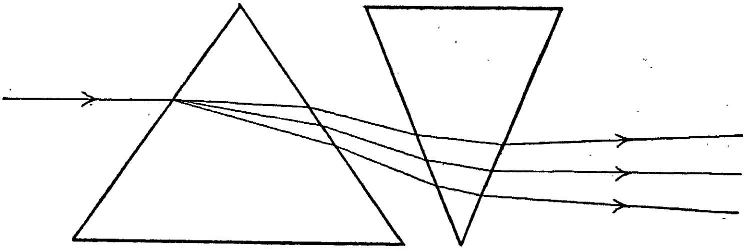

FIG. 77.

by which it is possible to see the spectrum in the direction in which the light is coming from the luminous body. The actual amount of separation of the colours is, of course, not so large as is obtained with the ordinary spectroscope, but it is sometimes very convenient to be able to look directly in the line of the source when a spectrum is being examined.

THE VELOCITY OF LIGHT.—The first estimation of the speed with which light travels was made by the astronomer Römer; he showed that the interval between successive eclipses of one of Jupiter's moons got gradually longer for a period of six months, after which the interval gradually got shorter. Römer rightly attributed this to the fact that light takes some time to travel from the satellite of Jupiter to the earth, and that if the observer was gradually travelling away from it, the interval between successive eclipses would get bigger, while if the observer on the earth were moving towards Jupiter, this interval would get shorter. By careful observation of these intervals over a period during which the earth traversed its whole orbit round the sun, Römer deduced that light travelled with the enormous speed of 190,000 miles per sec.

The next estimation of this speed also came from an astronomer, an Englishman, Bradley, who discovered a phenomenon to which he gave the name " aberration." If we picture a telescope pointed to any particular star in the heavens, the rays of light from the star enter the object glass and pass down the telescope, forming an image which is dealt with by the eye-piece in the way already explained. The telescope will, however, be moving through space with the speed of the earth in its orbit, so that unless the rays of light have an infinite speed, the image of the star will be displaced to an extent depending on the two speeds in question. Actually, Bradley found that a star near the pole of the earth's orbit appears to describe an ellipse about its mean position in the course of a year. The angle of aberration was not as much as a sixtieth of a degree; but such was the precision of Bradley's work that the speed of light that he deduced, viz., $3{\cdot}05 \times 10^{10}$ cm. per sec., is extremely near to the value now accepted.

The first successful attempt at measuring the speed of light by laboratory methods was made by the French physicist Fizeau. The principle of the method he used is as follows :—

Suppose that light is sent through one of the gaps in a spinning cog wheel and is sent back by a distant reflector just in time to be met by one of the cogs, then an observer on the other side of the wheel will see no light coming back through the cog wheel. The arrangement adopted by Fizeau to test this was similar to that shown in Fig. 78, and the results completely vindicated the claims of the astronomers that light travels with an enormous speed.

Light from the source S is reflected from a half-silvered mirror M, so that it passes through a gap in the cog wheel C, is then collected by the lens L_1, and sent as a parallel beam to L_2, which converges it to a mirror

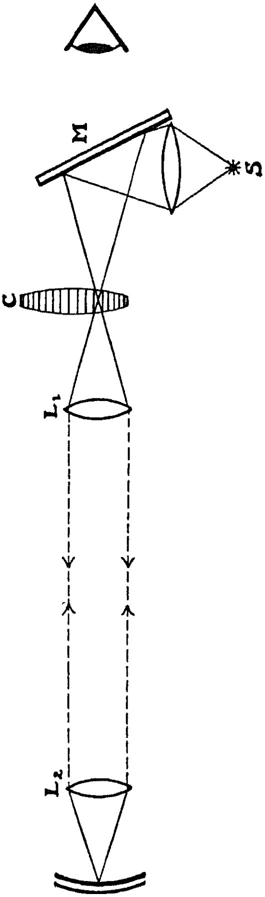

FIG. 78.

from which it is sent back along its path; a part of this reflected light, if not intercepted by one of the cogs of the wheel, reaches the eye of an observer E. The cog wheel in Fizeau's experiment had 720 teeth and revolved 12·6 times per sec. The path traversed by the light was 8,633 metres and the speed of light found by Fizeau was 295,000 km. per sec.

A few years later, Foucault devised a method for comparing the speed with which light travels through air and water. He found that it went 1·34 times quicker through air than water; reference to p. 87 shows that the refractive index of a ray of light in going from air to water is also 1·34; so that a real physical significance is given to the number representing the refractive index by this important discovery of Foucault—the bigger the refractive index, the more slowly light travels through the medium.

A series of measurements of the speed of light made in 1926 gave the result 299,796 km. per sec., so that very little error is made if we take the value 3×10^{10} cm. per sec.

THE POLARISATION OF LIGHT.—When light goes from a source A to B it is usual to say that the vibrations are at right angles to the direction in which the light goes. The vibration might be, as drawn in Fig. 79, in the plane of the paper; it might be perpendicular to this plane, or indeed make any angle with it, provided it was still at right angles to AB, and it is customary to think of the disturbance arising at A and going to B as quite unrestricted, in the sense that vibration is just as likely to occur in one plane as it is in any other, provided always that it is at right angles to the direction in which the light is travelling.

It is found, however, that certain substances have the power of restricting the vibration of light when it traverses them, and in these circumstances the light is said to be polarised.

One of the most striking instances of this is provided by the naturally occurring crystal tourmaline; these crystals are of a greenish colour and, when cut parallel to the crystalline axis, give crystals like A in Fig. 80. If a similar crystal be placed over A, but turned round through a right angle as in B, no light will go through the part where the two crystals overlap. This is explained by saying that tourmaline can polarise the light, restricting its vibrations so that they can only be executed in one plane; suppose, for example, up and down, but not from left to right. The effect of superposing B upon A is to extinguish all the light, because in the position indicated, the only vibrations that could get through B are not actually present in the light as it comes through A. Bartholinus discovered in 1669 that an object viewed through a crystal of Iceland spar appears to have two images, and to this phenomenon he gave the name double refraction; this is due to the crystal having one refractive index for the vibrations of light which are in one direction in relation to the structure of the crystal, and a different index for vibrations at right angles to this direction. In other words, a ray of light travels with different speeds through calcite according to the way in which the vibrations are orientated with respect to the structure of the crystal. A single ray is split up into two rays, one called the ordinary ray (having a refractive index of 1·658) and the other called the extraordinary ray (having a refractive index of 1·486).

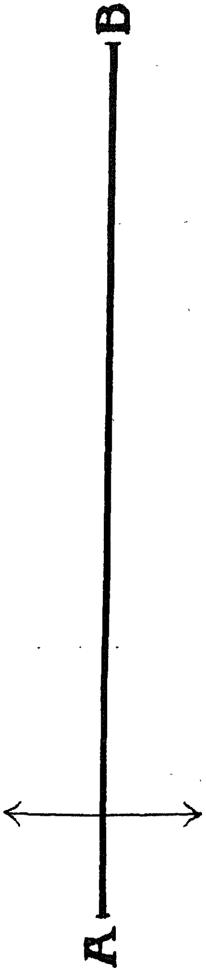

Fig. 79.

Nicol constructed a prism, which bears his name, in the following way: A piece of calcite is cut into two pieces, shaped as in Fig. 81, the two pieces then being cemented together by Canada balsam, which has a refractive index of 1·53. A ray incident at A is split up into two rays, the ordinary AO and the extraordinary

AE. When the ray AO reaches the interspace, it suffers total internal reflection, the other ray passing across this interspace and emerging on the far side. The emergent light is plane polarised, like the light which has passed through a crystal of tourmaline. Nicol proved that this was so by letting this emergent ray enter another similar prism rotated through 90° compared with the position of the first one; no light was transmitted through the second prism; it was, in fact, set to extinction. The function of the first prism is to polarise the light, and may be called the polariser; the second prism detects the state of polarisation, and is called the analyser.

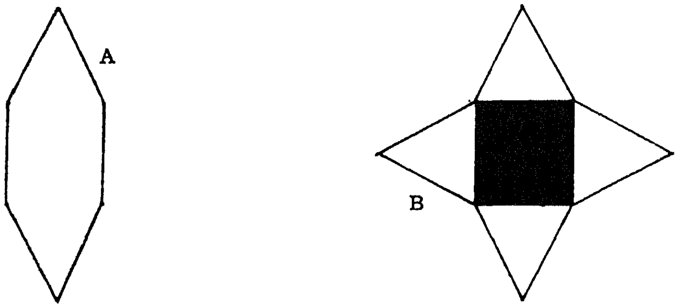

Fig. 80.

Arago discovered in 1811 that some substances, of which quartz is an outstanding example, have the power of rotating the plane of polarisation, so that if a Nicol polariser and analyser are set to extinction and a piece of suitably cut quartz is interposed between them, light passes through the analyser, the plane of polarisation of the light having been rotated; by finding the requisite angle of rotation of the analyser for extinction once more, the degree of rotation of the plane of polarisation can be found.

An instrument based upon this phenomenon is called a polarimeter, and the instrument devised by Laurent is very suitable for the examination of solids or liquids for finding their power of rotating the plane of polarisation of plane polarised light.

Light from a sodium flame is polarised by the Nicol prism P, the analysing Nicol A being mounted on the same axis in a tube which carries a small viewing telescope, as in Fig. 82. Initially P and A are set to extinction, but when an optically active body, *i.e.*, one which can rotate the plane of polarisation, is interposed, the field of view is no longer dark; extinction can, however, be brought about by rotating the analyser, and the extent of this rotation is measured on a circular scale.

It is not easy to say exactly when complete extinction of the light occurs, and a device known as a half-wave plate placed just behind the polariser, which splits the field viewed by the telescope into two parts, is often used. It is an easy matter to judge when these two parts of the field are equally illuminated, and this equality is upset by interposing the optically active body; the amount of rotation of the analyser necessary to get equality again measures the rotatory power of the body.

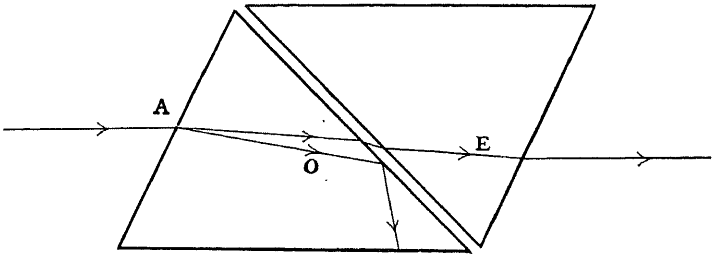

Fig. 81.

When use is made of definite quantities of optically active bodies, the polarimeter provides a very quick means of measurement, and it is used sometimes as a method for the estimation of sugars; dextrose and lævulose are chemically identical, though they disclose some structural difference by their power of rotating the plane of polarisation in exactly opposite directions.

It is found that the rotation of the plane of polarimeter occurs clockwise in some substances and anti-

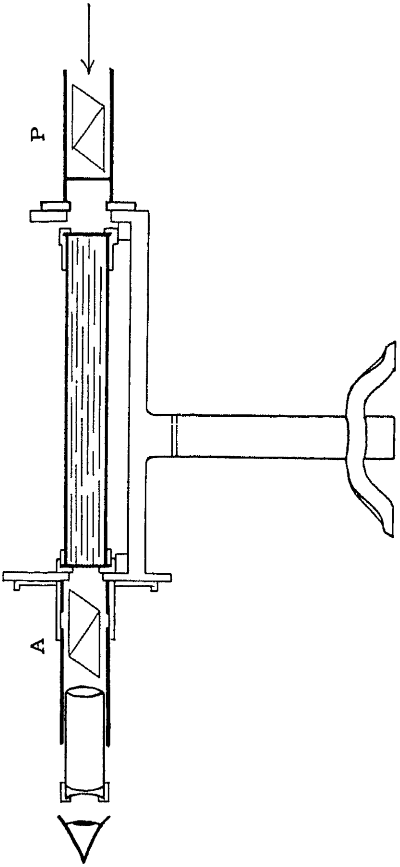

FIG. 82.

clockwise in others. The amount of rotation produced depends on the thickness of material interposed. Quartz has the power of rotating the plane of polarisation to an extraordinary degree, some samples being dextro-rotatory and others lævo-rotatory; in some samples it is as much as 21·72° per mm. thickness.

Looking along the ray of light towards the source of light, the rotation of the plane of polarisation is said to be right-handed if clockwise, but left-handed if anti-clockwise.

Biot stated the two following laws regarding the phenomenon :—

1. The amount of rotation is proportional to the thickness traversed by the ray.
2. The rotation produced by two substances is the algebraic sum of their separate rotations.

CHAPTER IV

SOUND

HEARING is one of the primary sensations, and when any sound is heard, it can be traced to some vibrating body. These vibrations are sometimes visible, as in the to and fro swing of the prongs of a tuning fork, but more often they are not visible. When a tuning fork is sounded, it vibrates in such a way that it sets the air surrounding it into vibration; these vibrations act upon the drum of the ear and are then transmitted to the internal mechanism of the ear, which is a very complicated structure; the final result is that some impression is conveyed to the brain, and the individual hears.

There is no standard sounding body which can be looked upon as giving a unit amount of sound per second in the same way as unit candle-power, or the calorie serve as units of illuminating power or heat.

Ordinary observation leads us to distinguish three main characteristics in any sound which we hear; they are—pitch or frequency of vibration; amplitude or loudness of the sound; and timbre or quality, which refers to the character of the sound. There are few people who would not at once recognise some essential difference between a single note played on a violin or banjo, or sung. The notes might have exactly the same frequency and loudness, but the timbre stands out as something distinct in each case. This difference is largely due to the fact that a vibrating body tends to set parts of itself or other neighbouring bodies vibrating, and these added vibrations give the note its quality. What makes a Stradivarius violin so highly prized is very largely a matter of these added vibrations which are evoked by the simple process of bowing the strings; or in the case of the human voice, the possessor of a voice like Caruso's is distinguished not by any special quality of vocal cords, but by an abundance of vibrations added to any note that is sung. These additional vibrations are called overtones, or harmonics. They can be studied very conveniently by stretching a metal wire between two points on a sounding board, a device known as a monochord. If a tuning fork in vibration is held with its stem on the sounding board, the wire is thrown into vibration if the force stretching the wire is adjusted to make the wire in tune with the fork. If a small paper stirrup is placed at the mid-point of the wire, the vibrations may be strong enough to jerk it off. When this happens, the stretched wire vibrates to and fro most strongly at the middle (anti-node) and not at all at the ends (the nodes) where the wire is clamped; it is then said to be giving its fundamental note. But the wire is at the same time giving overtones, and this may be shown by clamping the wire at its middle point, keeping all the other conditions the same. If the tuning fork is now held once again on the sounding board, it is found to throw the two segments of the wire into vibration with double the frequency of itself; this can be proved by putting the paper rider on to the middle of one of the segments and seeing that it vibrates; a further test shows that the note given out by each segment is in tune with a tuning fork of twice the frequency of the original one giving the fundamental note. Again, if the stretched wire is split up into three segments, each one of them will be found to vibrate when the original tuning fork is placed on the sounding board, and this time the pitch will be three times that of the fundamental note, and so on for higher frequencies. When the wire is vibrating freely without any constraints, all these overtones may accompany the fundamental note.

Experimentally it is found that the frequency of vibration n of a stretched wire can be expressed as follows :—

$$n=\frac{1}{2l}\sqrt{\frac{F}{m}},$$

where l is the length of the wire, F the stretching force, and m the mass per unit length of the wire.

A wire, 50 cm. long and weighing ·5 grm., *i.e.*, ·01 grm. per cm., stretched by a force of 1,000,000 dynes, would have a frequency or pitch of 100. The note c′ of a piano has a frequency of 256. Provided a monochord can be tuned accurately to a tuning fork, it gives a means of finding the frequency of the latter.

A more direct way, which also depends on tuning, is by using a siren which is made to sound in tune with the fork. The siren consists of a metal box fitted with a lid having a number of fine holes drilled in its periphery; a metal disc with similar holes in it sits on the lid of the box and is spun by means of air pressure; the jets of air in rapid succession produce a note which can be adjusted to be in tune with the fork. A counting mechanism on the disc gives the frequency of the note.

A second method, known as the falling plate method, may be used; in this case a light pointer is fixed to one prong of the tuning fork and adjusted so as just to touch a smoked glass plate held vertically by cotton threads, and in such a way that the lateral vibrations of the fork register horizontally on the plate. When accurately adjusted, the fork is set vibrating and the cotton suspension is burnt; the plate falls with gravitational acceleration and the to and fro motion of the prong is scratched on the plate as a wavy line. From such records the frequency of the fork can be found.

If two forks having very similar frequencies are sounded, beats are heard and the number of beats is equal to the difference in the frequencies. If they are frequent enough, the beats are not unpleasant, but when amounting to about 12 or 14 per sec. they give a very unpleasant sound; two forks exactly in tune can be made to give 4 or 5 beats per sec. by sticking a little wax on the end of one of the prongs of the forks.

THE DOPPLER EFFECT.—If a vibrating body moves towards or away from an observer, the note it emits appears to be altered in pitch. A familiar example of this is given by the whistle of an engine which, approaching one at high speed, gives out a note which is shriller than the actual frequency of the whistle; the pitch drops suddenly as the engine passes the observer.

If O be the observer and C the source of sound having a frequency n and a velocity v towards O, then it will be seen from Fig. 83 that the effect of this motion is to crowd the vibrations up in front of the moving source and thin them out behind it, so that there is an apparent shortening of the wave-length of the disturbances reaching O in the proportion $\left(\frac{V-v}{V}\right)$, where V is the velocity of the sound in air.

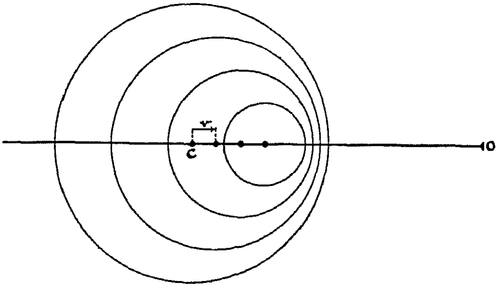

FIG. 83.

The effect on the frequency n is to increase it in the ratio $n\left(\frac{V}{V-v}\right)$.

If the real frequency of the whistle of an engine going at 60 m.p.h. is 1,000, the apparent frequency to an observer in the line of travel would be

$$n' = n\frac{V}{V-v} = \frac{1000 \times 1080}{1080-88} = 1089.$$

The Doppler principle is an important one and has applications in optics as well as in sound.

RESONANCE.—A column of air enclosed in a glass tube or metal pipe, as in Fig. 84, can be made to resonate to a vibrating body, such as a tuning fork, held near to it, in a very definite manner.

Suppose that the prong A of a tuning fork moves downwards; this will cause a wave of compression to go down the tube which will be reflected from the water interface. If by the time this reflection reaches the open end, the prong A just begins to move in an upward direction, a wave of rarefication moves down the tube, and this is reflected as before, and reaches the open end just as the prong A is ready to move downward once more and the cycle of operations is repeated again. In this way what are known as stationary vibrations are set up in the resonance tube, and if the length of the air column has been carefully adjusted to the frequency of the fork, a considerable reinforcement of the note is heard when the fork is held over the mouth of the tube. The velocity of a disturbance like sound is given by multiplying the frequency of the vibrating medium into the wave-length of the disturbance. This amounts almost to a hypothesis, and is also made use of in the transmission of waves, such as those over water or through the crust of the earth. Hence, if V is the velocity, n the frequency, and λ the wave-length of the disturbance, $V = n\lambda$.

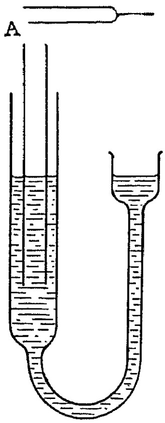

FIG. 84.

In Fig. 85 the wave-length is represented as the distance from one wave-crest to another; equally well it is the distance from one hollow to the next.

In the resonance tube described, the length of the tube is one quarter the wave-length of the vibration, so that if the frequency of the fork is known, the experiment provides an easy way of finding the velocity of sound, for if l is the length of the tube, we may write—

$$V = n\lambda = 4nl.$$

It is found that slightly different results are got by using tubes of different diameters, and the effective vibratory length of the resonance tube is $l + \cdot 6r$, where r is the radius of the tube.

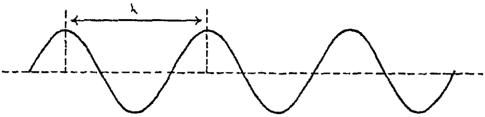

Fig. 85.

In an exactly analogous way to a vibrating wire, such a resonance tube can give overtones, and the simplest overtones of a closed pipe will be as shown in Figs. 86 and 87. If the pipe is open at both ends, it may still be thrown into resonance; and when giving its fundamental note, there will be an anti-node at each end (Fig. 88), for the air is equally free to vibrate at either end; in this case the node will be at the middle of the tube. The first overtone will be as shown in Fig. 89. If l be the length of the open tube, then for the fundamental note $l = \frac{1}{2}\lambda$, where λ is the wave-length of the sound. For the first overtone

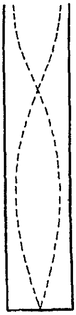

Fig. 86.

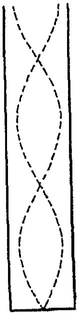

Fig. 87.

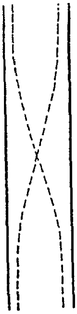

Fig. 88.

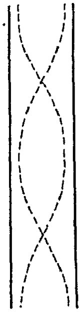

Fig. 89.

it will be seen that $l = \lambda$, and so on. It is not an easy matter to determine the length of these vibrating columns of air giving the best conditions of resonance, so that even if the frequency of the fork be known exactly, it is not possible to get very accurate values for the velocity of sound. This can, however, be done by observations out of doors, and measurements have been made by Colladon and Sturm of the velocity of sound through air and water. These were carried out on Lake Geneva by observers in boats about 8 miles apart. A gun was fired from one of the boats, and the observers in the other noted the interval between seeing the flash and hearing the sound. A bell was made to sound under water by a mechanism attached to the gun, and the sound waves travelling through the water were picked up by the observers by means of a trumpet-shaped receiver let down into the water over the side of the boat. These observations showed that sound travels much quicker through water than through air. The speed in air at 0° C. is 332 metres per sec., and through water it is about 1,430 metres per sec.

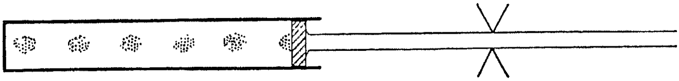

FIG. 90.

A very convenient method for comparing the velocity of sound in solids and in air has been devised by Kundt. The solid in the form of a rod is clamped in the middle, as in Fig. 90, and one end fits loosely into the open end of a glass tube, the other end of which is closed. The tube is thoroughly dried and a small quantity of lycopodium powder put into it, so that it forms a thin layer down the length of the tube. When the rod is stroked along its length with a resined duster, it is thrown into longitudinal vibration, and its fundamental note will be given when the middle of it is a node and the two ends anti-nodes; the vibrating rod throws the air into vibration by resonance, and the fine powder particles are heaped up at certain places (anti-nodes) along the tube. When the rod is sounded, the dust particles may be seen to be thrown into violent motion, and the middle of the vibrating parts may be taken as the anti-nodes; the distance between the anti-nodes is one-half the wave-length of the sound in air, and the length of the rod is half the wave-length of the sound in the rod for a note of the same frequency.

$$\therefore \frac{\text{Velocity of sound in air}}{\text{Velocity of sound in rod}} = \frac{\text{Distance between anti-nodes in air}}{\text{Length of rod}}.$$

The velocity of sound in air depends upon the temperature but not upon the pressure.

It can be proved from first principles that the velocity of sound in air is given by the expression

$$V = \sqrt{\frac{\gamma P}{D}},$$

where γ is the ratio of the specific heats of air (*vide* p. 39), P is the pressure, and D the density. If the temperature is kept constant and the pressure increased or decreased, the density will increase or decrease to an exactly similar extent (Boyle's law); but if the temperature is changed, then a change in density may occur without any change in the pressure. Hence the temperature will have an effect on the speed of sound.

If V_0 is the velocity of sound in air at 0° C. and D_0 the density at this temperature, V_t the velocity and D_t the density at any other temperature t; then :—

$$V_0 = \sqrt{\frac{\gamma P}{D_0}}.$$

$$V_t = \sqrt{\frac{\gamma P}{D_t}}.$$

But by Charles's law—

$$D_0 = \text{Mass per unit volume at } 0^\circ = \frac{M}{V_0}$$

and $$V_t = V_0(1 + \alpha t),$$

where α = coefficient of expansion of the gas.

$$\therefore\ D_t = \frac{M}{V_0(1+\alpha t)} = \frac{D_0}{1+\alpha t} = \frac{T_0 D_0}{T}.$$

Hence

$$V_t = \sqrt{\frac{\gamma P}{D_0 \frac{T_0}{T}}} = V_0\sqrt{\frac{T}{T_0}};\ \text{or,}\ \frac{V_t}{V_0} = \sqrt{\frac{T}{T_0}},$$

i.e., the velocity of propagation at any temperature t is to the velocity at 0° as the square root of the ratio of their absolute temperatures.

CHAPTER V

ELECTRO-STATICS

The study of electricity began with the discovery of the fact that, when certain things like amber are rubbed with silk, they have the power of attracting very light bodies to them. This property of amber was known as long ago as 600 B.C., but little interest was taken in the phenomenon till Dr Gilbert, physician to Queen Elizabeth, studied it in greater detail. He classed electricities as being either vitreous or resinous, and invented the name electrification to denote this particular state of such bodies.

A few experiments with very light objects such as two pith balls suspended by long silk fibres are enough to show that, when the two pith balls are touched with a glass rod which has been stroked with a piece of silk, they repel each other, and that they also do so when touched with an ebonite rod which has been rubbed with fur; but if one pith ball be charged by the glass rod and the other pith ball by the ebonite, then they attract each other. From this it is inferred that there are two different kinds of electrification; one is called positive and the other negative. If the glass rod and the piece of silk are tested separately, they are found to exhibit positive and negative electrification respectively; so that by the simple act of rubbing the glass with the silk, these two kinds of electricity are separated out on their surfaces. If we take a metal rod and try to do the same thing, no charge will be found on the rod; but if such a rod be mounted on a glass handle, then a charge can be developed on it by friction with a piece of silk. In the first instance, the charge is no doubt produced, but it is conducted away through the body of the performer to the earth and is so lost. This leads to the further classification of bodies as either conductors or insulators. All metals from this point of view would be called good conductors, while things like amber, sulphur, ebonite, sealing wax are good insulators. Things like marble, wood, or paper are neither good conductors nor good insulators.

Coulomb tried to find quantitatively how electrical charges attract or repel one another, and in this he succeeded in formulating the law which is often known as Coulomb's Law. He found that the force of attraction between two opposite charges Q and Q′, separated a distance *d* from one another, depended directly on the product of their charges and inversely as the square of the distance *d*. This may be written

$$F = \frac{QQ'}{d^2}.$$

It is a remarkable fact that Coulomb discovered this law, because the torsion balance which he used is not really accurate enough to establish the matter exactly, yet Coulomb got the correct result. In practice, electrical charges cannot conveniently be collected and used at points, but it is found that if we have a small insulated metal sphere and give it a charge, although this spreads itself uniformly over the surface, it behaves towards any charge outside it as if it were concentrated at the centre of the sphere, so that when two charged spheres attract or repel each other, the effective distance between them is the distance between their centres. Coulomb found that exactly the same law held for repulsion as for attraction between electrical charges.

The expression $F = \frac{QQ'}{d^2}$ gives us a simple means of defining a unit charge. If two equal and similar charges —when held 1 cm. apart—repel each other with unit force, *i.e.*, 1 dyne, then each charge is said to be a unit charge of electricity. If two equal and similar charges, 1 cm. apart, repelled each other with a force of 100 dynes, then the value of each charge would be 10 units, and so on.

About fifty years after Coulomb's work, Michael Faraday began a series of experimental researches on electricity which proved to be of epoch-making importance. Faraday, in his study of electro-statics, was anxious to find some rational explanation of the attractions and repulsions of electrified bodies; for this purpose he supposed that the real seat of action was not so much on the surfaces of the charged bodies as in the medium surrounding them. This medium he called the electrical field and supposed that it was traversed by tubes of electrical force, sometimes referred to as lines of force. He looked upon a positively charged body simply as the starting point of these tubes of force, and a negatively charged body as the place where they ended. In order to give a rational explanation of the electrical phenomena known to him, Faraday endowed these tubes of force with two properties; he supposed that they tend to contract along their length, so that they always get as short as possible and that they tend to repel each other laterally.

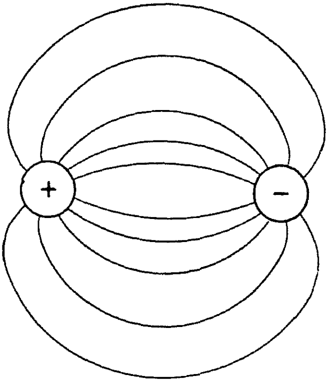

FIG. 91.

An insulated sphere, having a positive charge, is pictured as having one end of the tubes of force attache to it, the other ends reaching out indefinitely int space. The bigger the charge, the more numerous th tubes of force. If two insulated spheres are given equa and opposite charges, their mutual attraction is o these views explained by the contractile property of th

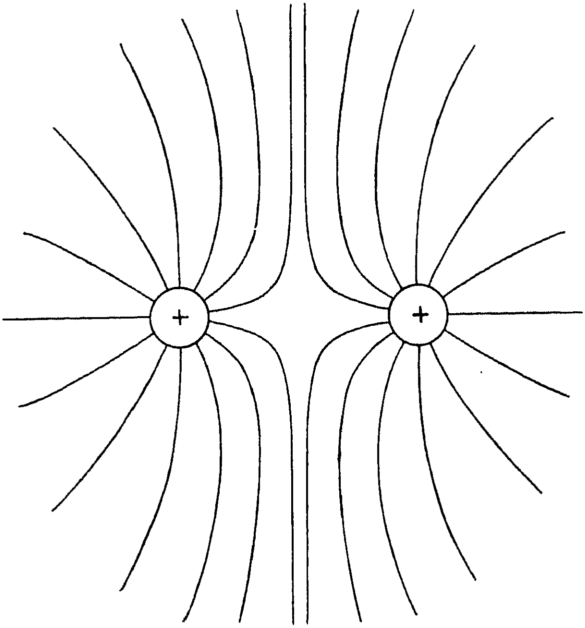

Fig. 92.

tubes, for instead of the tubes of force from the positivel charged body spreading out into space, they tend to g to the negatively charged body and are only stoppe from all crowding along the shortest path between th two, because of the lateral repulsion which the tube suffer. So that the tubes of force get bowed out as i Fig. 91, always leaving the surface of the conductor a right angles to it.

A similar kind of explanation can be invoked in th case of the repulsion of similarly charged bodies. Her the tubes of force issuing from one of the bodies cannot end on the other, and an asymmetry of the tubes as shown in Fig. 92 will give the effect of repulsion between two bodies similarly charged with electricity. This conception of Faraday's has been found very valuable, and no better explanation has yet been given of these fundamental phenomena.

Faraday showed that the force between two charged bodies is modified by changing the medium between them; in fact, it was this discovery that led him to suggest that the real seat of electrical action was in the space surrounding the charges.

The biggest force between two charges, a definite distance apart, is given when they are in a vacuum; the effect of material media is always to lessen the force, just as optically the effect of any medium is always to make light travel more slowly through it than through a vacuum. Media differ enormously in their power of diminishing the force acting along the Faraday tubes; air makes very little difference, whereas pure water diminishes it as much as eighty times. Faraday measured this effect for a number of substances, and suggested the term specific inductive capacity to denote how much the contractile force along the tubes of force is diminished. Some of these values are given in Table 6. The term dielectric constant is sometimes used for this quantity, but it is better to use the term that Faraday suggested.

Owing to the influence of the medium, Coulomb's law should be written :—

$$F=\frac{1}{K}\cdot\frac{QQ'}{d^2},$$

where K is the specific inductive capacity of the medium. Faraday not only showed that the action of material media was to alter profoundly the force acting in the regions permeated by tubes of force, but by a series of experimental researches he found that an electrically charged body has the power of inducing a charge of opposite sign on a neighbouring body. This can be shown very simply by means of a gold leaf electroscop no metal can be rolled into such thin sheets as gol and if two strips of gold leaf are hung close to o another from a vertical brass rod, they are very sensiti to electrical charges; if the leaves are given simil charges, they are so light that the electrical force repulsion is able to make the leaves diverge, and t extent of this divergence can be measured with well-ma instruments and made to serve as an indicator of t electrical charge given to the leaves.

TABLE 6

Substance.	Value of Specific Inductive Capacity.
Air	1·00058
Water . . .	80
Paraffin . . .	2 to 2·3
Ebonite . . .	2·8
Sulphur . . .	3·6 to 4·3
Mica	5·7 to 7·0

Suppose that an ebonite rod charged negatively friction is held over the top of an electroscope, such that shown in Fig. 93. The tubes of force ending the top plate of the electroscope will be positive, ar this positive charge is thought of as being held by tl inducing positive charge; the leaves diverge with neg tive electricity, the positive ends of the tubes of for being on the inside of the instrument. This negati charge is to be looked upon as a free charge, and if tl underside of the top plate be touched with the finge the charge readily vanishes to earth, the gold leav collapsing; the positive charge remains held by the negative charge on the rod, but if the rod is taken away, the positive charge becomes free and the leaves will diverge once more, this time with positive electricity. The experiment may be varied in many ways, and Faraday showed that a charge of electricity was capable not only of inducing an opposite charge on neighbouring bodies, but that the amount of this induced charge is exactly equal to the inducing charge. If all the tubes of force from the charged body end on a single other body, this body will receive the whole of the induced charge.

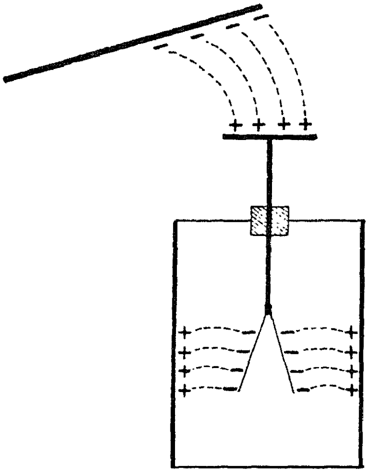

Fig. 93.

Faraday gave the name electro-static induction to this phenomenon.

The electrical potential at any point in an electrical field is the amount of work which would have to be done in order to bring a unit positive charge up from infinity to the point in question. It can be proved that, if a positive charge Q be at a point, then in order to bring a unit positive charge up from infinity to within a distance d_1 of it, the work done is $\frac{Q}{d_1}$, to within a distance d_2 it would be $\frac{Q}{d_2}$, and so on. Hence the would be a difference of electrical potential between the two points, amounting to $\frac{Q}{d_1} - \frac{Q}{d_2}$, and it is this differen which determines which way positive electricity w move if nothing else is hindering it.

The proof of the above result is as follows :—

Let A and B be two points distant a and b from positive charge Q. The force on unit charge at A $\frac{Q}{a^2}$ and at B is $\frac{Q}{b^2}$. Let points A_1, A_2, A_3 in between and B be at distances a_1, a_2, a_3 from Q. The wo done in moving unit charge from A to A_1 is the avera force operating over this distance multiplied by t distance; the force at A is $\frac{Q}{a^2}$, that at A_1 is $\frac{Q}{a_1{}^2}$, and t average force is $\frac{Q}{aa_1}$, so that the work done in goi from A to A_1 is $\frac{Q}{aa_1}(a_1 - a) = \frac{Q}{a} - \frac{Q}{a_1}$.

Similarly for the next step, the work done in movi the unit charge from A_1 to A_2 will be given by $\frac{Q}{a_1} -$ and for the next step it will be $\frac{Q}{a_2} - \frac{Q}{a_3}$, and so on. T total work done by the electrical forces in moving ur charge from A to B will be the sum of all these quantitie on adding these up it will be found that all intermedia quantities cancel, leaving $\frac{Q}{a} - \frac{Q}{b}$.

A point is said to be at a higher electrical potenti than another if positive electricity tends naturally travel from it to the other point.

Unit difference of potential exists between tv points if unit amount of work (*i.e.*, 1 erg) is done taking a unit positive charge from one point to the other.

Suppose that a number of metal spheres of radii 1, 2, 3, and 4 cm. are fixed in insulating stands and then all given exactly the same charge of positive electricity. This charge will spread uniformly over each of the spheres, but it will be found that the electrical potential of these four spheres will be in the ratio of 4, 3, 2, 1, *i.e.*, inversely as their radii, for the smallest sphere is raised to the highest potential. From this simple type of experiment and many others, it has been found that the electrical potential acquired by a body when it is given a charge depends upon some quantity called the "electrical capacity" of the body. This capacity depends very largely on the geometry of the body; for instance, for a sphere it is proportional to its radius; a sphere of 1 cm. radius is said to have unit capacity, one of 10 cm. radius has 10 units of capacity, and so on. It will be noticed that the electrical capacity is not proportional to the volume of the body, for the volumes of these two spheres are in the ratio of 1 to 1,000. If one sphere of radius *a* (Fig. 94) be

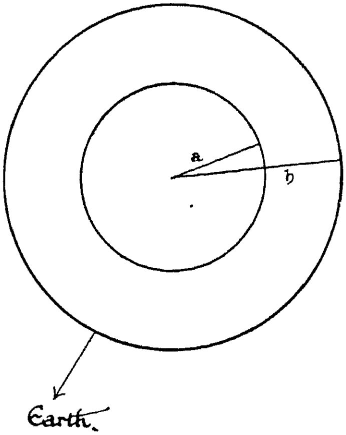

FIG. 94.

surrounded by a hollow one of radius b, then the electrical capacity of the two together is $\frac{ab}{b-a}$. The proof of this need not be gone into here, but it should be noticed that, if the gap between the spheres is made very small, then $\frac{ab}{b-a}$ becomes very large, although neither a nor b is large; in other words, a very large electrical capacity can be got within a comparatively small volume. The name condenser is often applied to an electrical capacity, and one of the most usual forms of condenser is known as the parallel plate condenser. In its simplest form this consists of two parallel plates of metal kept apart by some insulating material, such as glass, solid paraffin, or mica. If K is the specific inductive capacity of the insulating medium, A the area of the plates and d the distance they are apart, the electrical capacity of such a condenser is $\frac{KA}{4\pi d}$.

If $K=3$, A the area of either plate $=1$ sq. metre, $d=1$ mm., then the electrical capacity will be

$$\frac{3\times10^4}{4\pi\times\cdot1\text{ cm.}}=3400\text{ cm.}$$

In order to get such a capacity by using a single sphere, we should have to have one of 34 metres radius, obviously a very inconvenient size. In fact, a single parallel plate condenser with plates as large as a square metre would be unnecessarily spread out, for exactly the same result could be got by having a number of smaller plates coupled up alternately to positive and negative terminals. This is, in fact, the usual form condensers take. Thin layers of tin or lead foil separated by sheets of paper soaked in paraffin wax make very useful condensers, and relatively enormous capacities can be got by this means into a very small compass.

If Q be the electrical charge given to a capacity C, then the electrical potential V to which it is raised will be given by the simple relation,

$$V = \frac{Q}{C}.$$

This may, of course, also be written

$$C = \frac{Q}{V}; \text{ or, } \quad Q = VC.$$

An electrical system has unit capacity if unit charge given to it raises its potential by unity.

The unit charge, unit difference of potential, and unit capacity which have been defined are found in practice to be very inconvenient units, so other units called practical units have been set up for this purpose. These units have all been named after scientists who have made an important contribution to the advancement of knowledge in electricity or magnetism. The three practical units of charge, difference of potential and capacity are called the coulomb, the volt (volta), and the farad (faraday), and the relation that they bear to the theoretical units may be seen from the data in Table 7 :—

TABLE 7

	Practical Units.	Theoretical C.G.S. Electro-static Units.
Electrical charge . .	1 coulomb =	$10^{-1}v$
Difference of potential .	1 volt =	$10^8 \div v$
Electrical capacity .	1 Farad =	$10^{-9}v^2$

v is the speed of light, viz., 3×10^{10} cm. per sec.

The theoretical units which have so far been mentioned are known as C.G.S. electro-static units, because they have been established on the basis of centimetre, gram, second units from an electro-static point of view. It will be seen later on, that other theoretical units of these same three quantities, viz., charge, potential difference, and capacity, have been established also on the basis of C.G.S. units, but from an electro-magnetic point of view. These different units often constitute a real difficulty for those beginning the study of electricity, but the solution of a few numerical problems and some practical experience with electrical quantities soon reconcile one to their use.

The practical units were decided upon by a Committee of the British Association in 1874. Copies of the standards then set up are available all over the world.

In practice, the electrical charge on a body is very rarely measured directly, for it is found more convenient to measure the other two quantities V and C, the product of which will give Q.

The practical unit of difference of potential is 1 volt, and instruments designed to measure volts are called voltmeters. These take a variety of forms, some of which will be met later on in current electricity; but an electro-static voltmeter is one which, when in action, does not allow a current of electricity to pass through it. A good type of voltmeter is shown in Fig. 95. A set of movable vanes is mounted on a vertical spindle so as to engage with another set of fixed vanes; if the two sets of vanes are given opposite charges, the movable ones are attracted into the gaps between the others; the extent of this movement is controlled by a spring attached to the vertical spindle. The volts are read directly from the deflection of an indicator (also attached to the vertical spindle) over a calibrated scale; instruments of this sort are very useful for measuring voltages varying from about 50 to 50,000 volts, but are unsuitable for lower voltages.

Lord Kelvin invented the quadrant electrometer, which consists of a flat metal needle of rather big area suspended in a shallow box split up into four insulated quadrants, the opposite quadrants being connected together electrically, as shown in Fig. 96. If the needle be given a positive charge and suspended symmetrically between the quadrants, then it will move into the set of quadrants which is charged negatively. With a very light needle and a fine suspension of quartz, there is little difficulty in measuring as little as $\frac{1}{100}$ of a volt difference of potential. Any very small movement of the needle is shown by Kelvin's device of mounting a very small mirror on the vertical support of the needle. A beam of light is directed on to this and the reflected beam forms an image on a distant scale. If the needle moves, the mirror goes with it and the "spot of light" moves over the scale. Once the scale is calibrated, the voltage corresponding to any deflection is read off.

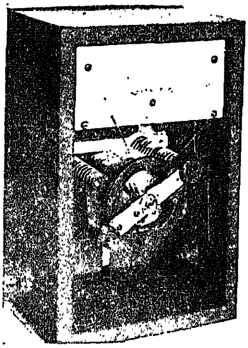

FIG. 95.

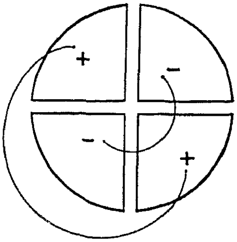

FIG. 96.

The design of quadrant electrometers has been very much perfected in recent years, and they are now largely used for very delicate measurements.

The measurement of an electrical capacity is nearly always made by comparing the unknown capacity with a standard one. If both of them are charged up to the same difference of potential, then they will have charges on them directly proportional to their capacities. It is, however, very inconvenient to measure electro-static charges unless they are set into motion and produce electrical currents; so this is what is generally done, and by measuring the currents which the two capacities give when discharged through an instrument called a ballistic galvanometer, the two capacities can be compared in a very simple and direct way.

CAPACITIES IN PARALLEL OR SERIES.—Suppose that we have three condensers of capacity, C_1, C_2, and C_3. If they are connected as in Fig. 97, so that they all have a common positive and negative boundary, they are said to be connected in parallel.

If $V = V_1 - V_2$, *i.e.*, the common difference of potential, then the charges on the condensers will be C_1V, C_2V, C_3V. It is obvious that these three condensers could

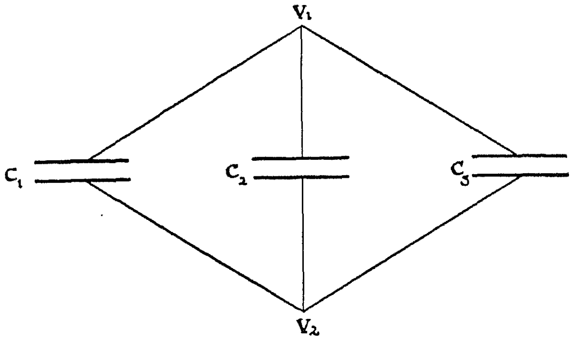

FIG. 97.

be replaced by a single one of suitable capacity. Let its capacity be C, then if this were joined up in the circuit replacing the others, it would have a charge CV. This charge must be the same as the total charge of the individual condensers which it is displacing. Hence,

$$CV = C_1V + C_2V + C_3V;$$

or,

$$C = C_1 + C_2 + C_3.$$

Hence the capacity of any number of condensers joined in parallel is equal to the sum of the individual capacities.

Suppose now that they are joined in series, as shown in Fig. 98. In this case, although the condensers are all of different capacity, they get the same charge; this may most easily be seen by thinking of Faraday's tubes as stretching from the top plate to the bottom and being intersected by the other surfaces; whatever these may do to the tubes of force, they cannot alter their number, so that the charge on each surface is the same numerically. Hence if this charge is Q,

$$Q = C_1(V_1 - V_2) = C_2(V_2 - V_3) = C_3(V_3 - V_4) = C(V_1 - V_4),$$

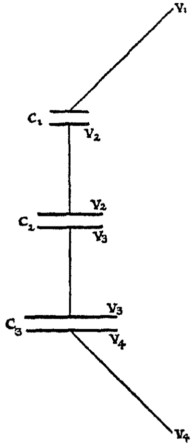

Fig. 98.

where C is the equivalent capacity of the arrangement,

or,
$$V_1 - V_2 = \frac{Q}{C_1}.$$

$$V_2 - V_3 = \frac{Q}{C_2}.$$

$$V_3 - V_4 = \frac{Q}{C_3}.$$

$$\therefore\ V_1 - V_4 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = \frac{Q}{C}.$$

$$\therefore\ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

The effect of arranging capacities in series, therefore, is to decrease the whole capacity; it is often done, however, in order to split up the voltage in a circuit.

When a condenser is connected to some source of potential difference, it acquires a charge proportional to its capacity and to the potential difference. If it is disconnected from the source, it retains its charge indefinitely if the insulation conditions are good. Suppose that, when it is insulated and holding a charge, the positive and negative terminals are connected; the condenser is in this way discharged, but it has been found that the discharge of a condenser may be rather a complicated event, the charge sometimes oscillating between the two boundaries of the condenser. Such a discharge is in fact called an oscillating discharge, but these oscillations occur so rapidly that, to the naked eye, the discharge appears to be an instantaneous process.

If a condenser of 1 microfarad capacity be charged by a source of 100 volts potential difference, the charge on it will be given by

$$Q = VC = 100 \times 10^{-6} \text{ coulombs} = 100 \text{ micro-coulombs.}$$

If it were short circuited by a thick copper wire bent into a circle of about 1 ft. radius, the charge would oscillate between the boundaries of the condenser at a rate of about 100,000 times per sec.

There are several ways in which the rate of oscillation can be varied, and it is possible by having enough resistance in circuit to arrange that a condenser discharges without any oscillations at all.

Electro-static Machines.—The development of electrical charges by friction led to the invention of machines by which this could be done rapidly, and by storing such charges considerable differences of potential could be obtained. These machines were used somewhat largely in medicine at a time when electricity was a rather unknown quantity and such machines were, it must be owned, often used because of their psychological rather than their physiological effects. An excellent account of these early friction and influence machines is given by Colwell in his " Essay on the History of Electrotherapy and Diagnosis," 1922.

At the present day they are very rarely used, as they have been very largely superseded by much more reliable instruments. The only one that need perhaps be mentioned here is Wimshurst's influence machine, a working model of which is still to be found in most physical laboratories; but a detailed description of it is omitted here, as it is thought that no purpose is served by the student acquiring details of a machine he is unlikely to come across, because more efficient appliances are now used.

CHAPTER VI

MAGNETISM

THE word magnetism is derived from Magnesia in Asia Minor, where naturally-occurring minerals have long been known which possess a very curious property. They are said to be magnetic, and lumps of this mineral have the property that, when they are hung up so that they can turn freely about a horizontal axis, they tend to turn in a certain direction; on this account they were known as lode stones or leading stones. This property seems to have been known for many centuries, and the Chinese are credited with using such lode stones in navigation, the modern equivalent being the navigator's compass.

The main constituent of these lode stones is iron, and although many other substances can be shown to have magnetic properties, iron (and steel, which is mainly iron) stands in a class by itself as being much more magnetic than any other substance known.

A magnetic substance, therefore, is one which itself exhibits magnetism or is susceptible to magnetic forces.

A magnet is a body which shows its magnetism in certain well-defined ways. An ordinary bar magnet has two regions called its poles where its magnetism is particularly displayed, and the space around the magnet may be shown to be permeated by a magnetic field. William Gilbert, at one time physician to Queen Elizabeth, discovered that the earth behaves as if it were a magnet. There is a region called the north magnetic pole and another region called the south magnetic pole. These poles do not coincide with the north and south geographical poles; in fact, the magnetic poles are not absolutely fixed in position, as will be explained in more detail later.

The two poles of a bar magnet are regions nearly, though not quite, at the ends of the magnet, and they are called north and south poles, the north being the pole which tends to point towards the north magnetic pole of the earth. Since it is found by experiment that the north pole of one bar magnet repels the north of another bar magnet, it would be more correct to call the pole of a magnet which is attracted to the north pole of the earth a south pole; it is sometimes called the north-seeking pole, and perhaps contraction of this term accounts for its being called a north pole.

Similar poles repel and, unlike poles, attract each other. By taking a long knitting needle and magnetising it, the two poles can be separated a considerable distance from one another, and Coulomb devised an experiment by which he could measure the force of attraction and repulsion between magnetic poles. He found that it varied directly as the strength of the poles and inversely as the square of the distance separating them. If m and m^1 are the strengths of two poles and d the distance separating them, then the force may be expressed as $F=\frac{mm^1}{d^2}$. The force is, however, found to depend upon the nature of the medium between the poles in very much the same way as the force between two electrical charges varies according to the medium in which the charges are placed. In the magnetic case this quantity is called the magnetic permeability μ, and Coulomb's law is then written

$$F=\frac{mm^1}{\mu d^2}.$$

If Coulomb's experiment is done in air, no appreciable error is made by putting $\mu=1$. A unit magnetic pole may be defined as that pole which, when placed 1 cm. away from another similar pole, repels it with unit force, *i.e.*, 1 dyne. A north pole of 10 units strength is one which would repel a unit north pole placed 1 cm. away from it with a force of 10 dynes, and so on.

Although iron is the chief magnetic substance in the earth, it is found that very few substances exist which are uninfluenced by magnetic forces. Substances are classified as paramagnetic and diamagnetic, according to whether magnetic lines of force tend to go through them more readily than through the air; or, on the other hand, whether the lines tend rather to avoid the medium. An outstanding example of a diamagnetic body is bismuth, but this property is not shown anything like as strongly in bismuth as paramagnetism is in iron or steel.

The phenomenon of magnetic induction is analogous to that of electro-static induction and can be shown in many ways.

An ordinary bar magnet will attract to itself ordinary iron nails which are magnetic though not necessarily magnets. The north pole of the magnet induces a south pole on the part of the nail nearest to it and a north pole appears at the part most remote, so that temporarily the nail becomes a small magnet. The south pole induced upon it is attracted to the pole of the magnet inducing it, and this force may be big enough to support the weight of the nail. If the magnet is strongly magnetised, it will support a long chain of nails. If the nails are shaken off, they are found to be magnets no longer. It is, in fact, very difficult to keep ordinary soft iron, though very magnetic, a permanent magnet, whereas steel magnets may retain their magnetism for years. A bar of steel may be made into a magnet by means of another magnet by the method of "stroking," but it is not usual to do it this way, as it can be done much more conveniently by means of electrical currents; how this is done will be considered in the section devoted to current electricity.

In Fig. 99, if $2l$ is the distance separating the poles of a bar magnet and m the strength of these poles, then $2ml$ is called the magnetic moment of the magnet and is written M. It is obvious that two magnets may have the same magnetic moment, yet different pole strengths; or, on the other hand, different magnetic moments though of similar pole strengths.

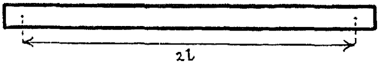

FIG. 99.

The magnetic field around a bar magnet can be mapped in a very simple way by putting the magnet on a table, laying a sheet of smooth paper on it and sprinkling iron filings over the paper; when the paper is gently tapped, the filings are seen to take up definite positions owing to the fact that they behave like little magnets and tend to set in the direction of the magnetic field. The distribution of the magnetic field surrounding ordinary magnets is seen in Figs. 100 to 102. Fig. 100 is an actual photograph of iron filings sprinkled round an ordinary bar magnet; they assume the direction

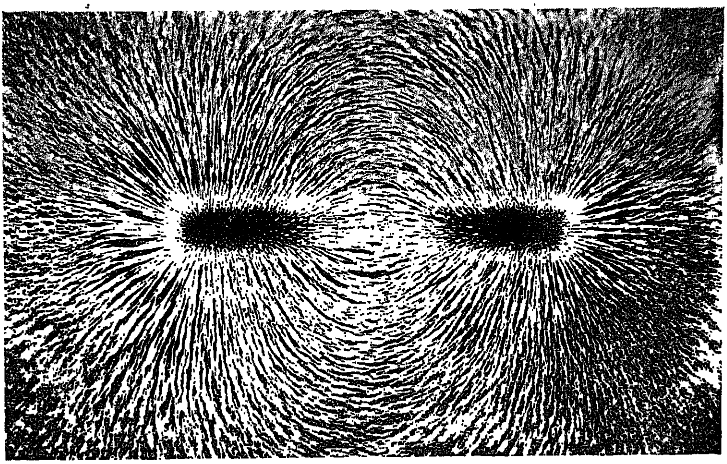

FIG. 100.

of the magnetic field. Fig. 101 represents in the same way the magnetic field between two unlike poles and Fig. 102 the field between two similar poles.

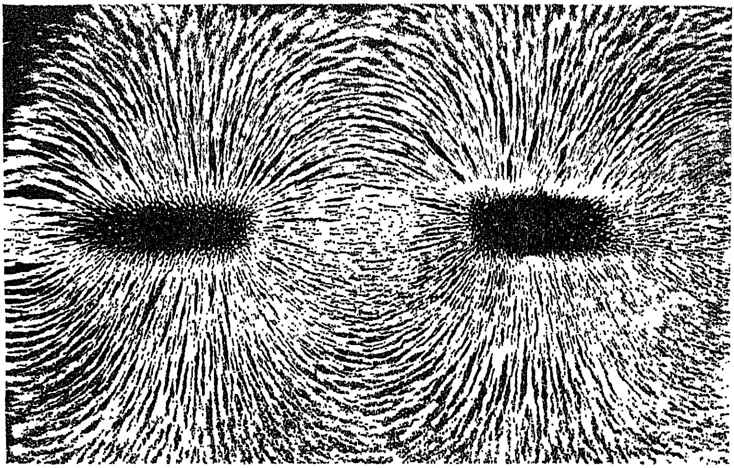

FIG. 101.

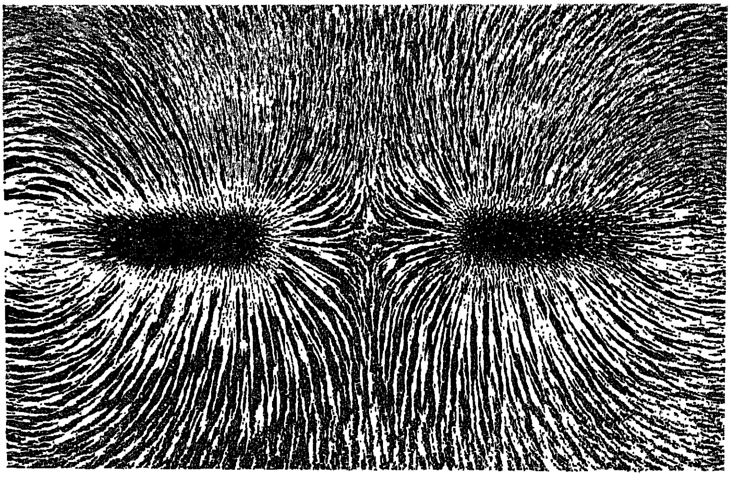

FIG. 102.

The intensity of a magnetic field is measured by the force experienced by a unit magnetic pole when it is put in the field. If the force on the unit magnetic pole is a dyne, the magnetic field at that point is said to have unit intensity, and this is called a gauss, *i.e.*, 1 dyne per unit pole. If a pole of 10 units pole strength is put at the same point, the mechanical force on it will be 10 dynes.

It is important to be able to find the way in which the magnetic force due to a bar magnet varies along its axes. The two positions generally chosen are called the end-on and the broadside positions.

End-on Position.—Let P in Fig. 103 be any point on the axis of a bar magnet of pole strength m, and let $2l$ be the distance between its n and s poles and d the distance of P from the centre of the magnet. Then the force on a unit north pole placed at P will be one of repulsion due to N and of attraction due to S. The resultant force will be

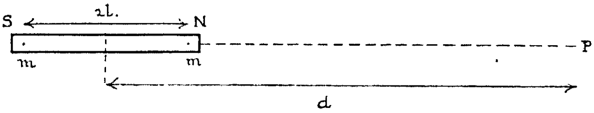

Fig. 103.

$$\frac{m}{(d-l)^2} - \frac{m}{(d+l)^2} = \frac{4mdl}{(d^2-l^2)^2}.$$

If d is large compared with l, l^2 may be neglected compared with d^2, so the resultant force

$$F = \frac{4ml}{d^3} = \frac{2M}{d^3}, \text{ for } M = 2lm.$$

Hence the force along the axis varies very rapidly as the distance from the centre of the magnet is increased.

Broadside Position.—Let P in Fig. 104 be any point distant d from the centre of the same bar magnet. A unit north pole put at P will be repelled by the north pole of the magnet with a force F_1 and attracted by the south pole with a force F_2. These forces are equal because the two poles of the magnet are of equal strength and they are equidistant from P. Their resultant will be given by F obtained by completing the parallelogram of which F_1 and F_2 form the two adjacent sides. The

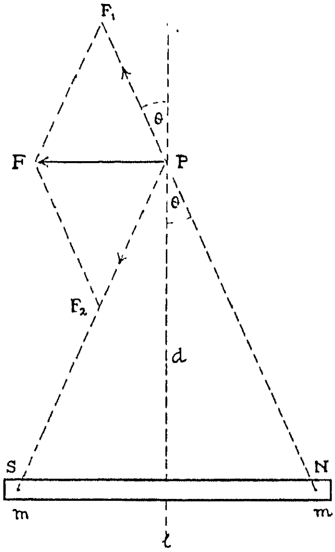

FIG. 104.

numerical value of F is given by adding the two horizontal components of F_1 and F_2 together.

$$F_1 \sin \theta = F_1 \times \frac{l}{\sqrt{d^2+l^2}}.$$

$$F_2 \sin \theta = F_2 \times \frac{l}{\sqrt{d^2+l^2}}, \text{ and } F_1 = F_2 = \frac{m}{d^2+l^2}.$$

$$\therefore\ F = \frac{2ml}{(d^2+l^2)^{3/2}}.$$

As before, if d is large compared with l, l^2 may b neglected compared with d^2.

$$\therefore\ F = \frac{2ml}{d^3} = \frac{M}{d^3}.$$

Comparing this with the previous expression, it will be seen that, provided the distance from the magnet is large compared with the length of the magnet, the force at right angles to the magnet is just one-half of what it is at an equal distance along the long axis; the force still varies, however, as the inverse cube of the distance from the centre of the magnet.

THE MAGNETOMETER.—A small magnetic needle pivoted so that it is free to move horizontally, takes up a definite position owing to the action of the earth's magnetic field. A vertical plane through the polar axis of this magnet is called the magnetic meridian. If a magnet of moment M is brought near to the needle, so that this is in the end-on or broadside position with respect to it, it will come under the influence of a magnetic field, the value of which has just been determined. Suppose that the small magnet is in the end-on position, the direction of the earth's horizontal field being at right angles to that of the other magnet, as in Fig. 105. Under the action of this magnet, the small magnetic needle rotates, say, through an angle θ. When it is in equilibrium,

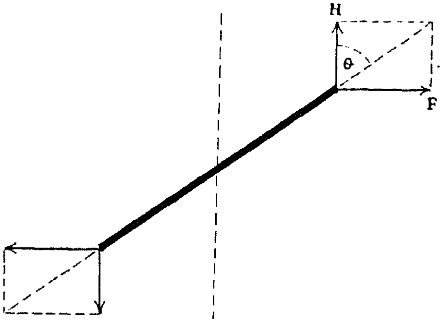

FIG. 105.

$$\frac{F}{H} = \tan\theta; \text{ or, } F = H\tan\theta.$$

But

$$F = \frac{2M}{d^3}.$$

$$\therefore \ \frac{M}{H} = \tfrac{1}{2} d^3 \tan \theta.$$

In this way a comparison can be made between the magnetic moment of any magnet and the strength of the horizontal field of the earth. But this is only a ratio and gives no determinable value for either M or H.

Another experiment with the same magnet, however, gives additional information, by means of which M and H can both be found.

If the magnet be suspended through its C.G. so that it can oscillate in a horizontal plane, it is found that the time of oscillation can be expressed by the formula

$$T = 2\pi \sqrt{\frac{I}{MH}},$$

where M = magnetic moment of the magnet, H = horizontal field of the earth, and I = moment of inertia of the magnet. The moment of inertia of a body depends upon its mass and its geometry with respect to the axis of rotation. For an ordinary bar magnet of mass *m* and semi-axes *a* and *b*, it can be calculated from the formula

$$\text{Moment of inertia} = m\left(\frac{a^2 + b^2}{12}\right).$$

Rewriting the formula, we have

$$MH = \frac{4\pi^2 I}{T^2}.$$

It will be seen that one set of observations gives $\frac{M}{H}$ and the other MH, so that combining the two, we may derive both M and H. These two experiments, therefore, are of fundamental importance in measurements in magnetism.

The magnetic moments of two magnets may most easily be compared by means of the first experiment, two sets of observations at the same place (H being constant) leading directly to

$$\frac{M_1}{M_2}=\frac{\tan \theta_1}{\tan \theta_2}.$$

A comparison can be made of the value of H at two different places by finding the time of oscillation of a magnet at the two positions. From the formula given, it can be deduced that

$$\frac{H_1}{H_2}=\frac{T_2^2}{T_1^2}.$$

Magnetic surveys are made on these lines.

The magnetism of the earth has been the subject of a great deal of experimental study and continues to be so, because it is found that the strength and direction of the magnetic field of the earth varies considerably not only from place to place, but also in the course of time. The causes of these irregularities are not completely known, but the variations are largely due to the fact that large masses of magnetic material (mostly iron) in the earth are not stationary, and their movements in and below the earth's crust so alter the distribution of the magnetic lines of force, that changes in the direction and intensity of the earth's field are always going on. Roughly speaking, the earth behaves like a single large magnet with its north and south poles not far from the geographical poles. The angle that a magnetic needle, free to move horizontally, makes with the geographical north is called the magnetic declination, and the angle it makes with the horizontal, if set so that it is free to move in a vertical plane, is called the angle of dip.

The instrument used to find this angle is called the dip circle, which consists of a magnet pivoted so that it can swing on its short axis in a vertical plane; a circle divided off into angular degrees is mounted beside it in a vertical plane also, so that the angle of dip can be read from it. In London, the angle of dip is about 67°

at the magnetic north it is of course 90°, and at the magnetic equator the needle does not dip at all.

The study of variations in the apparent magnetism of the earth is carried out chiefly in magnetic observatories, but a great deal of work also is done year by year in charting the seas, for the variations of the compass are vital matters in navigation.

MOLECULAR MAGNETISM.—If a bar magnet is broken in two, the two parts are found to behave as separate magnets, and if the separate parts are broken again, the same thing happens; it is in fact impossible to isolate a magnetic pole in this way, and this has led to the view that the magnetic property of bodies is essentially a molecular property. It is extraordinary that one substance, iron, is so much more magnetic than any other, but investigation has shown that very few substances indeed are indifferent to the action of strong magnetic forces, when these are brought to bear upon them. As will be explained later, it is possible to produce very strong magnetic fields by means of electrical currents, and the magnetic field between the poles of a strong horse-shoe shaped electro-magnet has often been used to test the effect of very strong magnetic fields upon substances which were previously thought to be non-magnetic. In practically every case it has been found that there is some action of the kind that would happen if there were some, though small, magnetic property in the bodies. The question then arises, what is the essential difference between an ordinary bar magnet and the same piece of iron before it has been magnetised? The molecular hypothesis is that the molecules of iron are potential magnets, but that in the unmagnetised condition of the iron bar, the molecules of iron are orientated quite at random. Now the distribution of the lines of force round the same bar of iron when magnetised strongly suggests that within the iron the molecular magnets are arranged in an orderly fashion, the north pole of the one facing the south pole of the other, along practically the whole length of the magnet, and the act of magnetisation, therefore, in all probability, consists not in actually magnetising the molecular magnets, but in getting them to set in this sort of way. How it is that this process is so much easier in the case of iron than in any other substance is, however, very obscure.

CHAPTER VII

CURRENT ELECTRICITY

It was discovered by Oersted in 1827, that when a current of electricity goes along a wire, a magnetic field is produced in the space surrounding the wire. He proved this by taking a compass needle and fixing a wire above the needle so that it lay in the same direction as the needle, which, of course, set in the direction of the earth's horizontal magnetic field. When an electric current was sent through the wire, the compass needle was deflected; the stronger the current and the nearer the wire was held to the needle, the greater the deflection. When the current was reversed in direction, the deflection of the needle was also reversed. Such a discovery

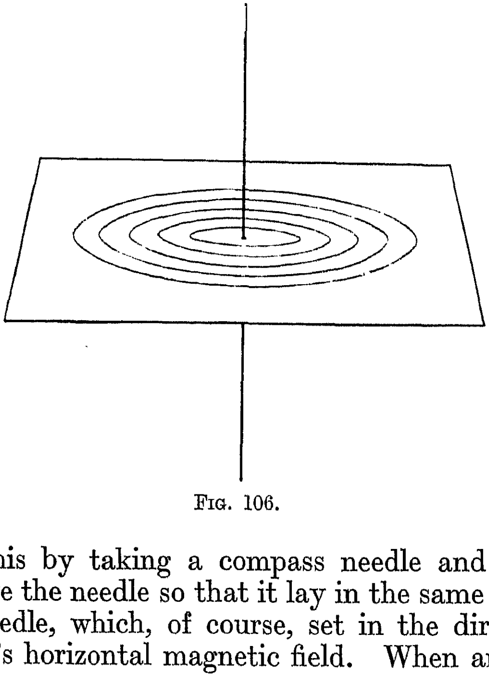

Fig. 106.

gave a great impetus to the study of current electricity, and we owe the early developments of the subject largely to the French physicist Ampère.

The magnetic field due to a current going along a straight copper wire can be shown by fixing a thick copper wire vertically through a piece of smooth white cardboard; if iron filings are lightly sprinkled over the cardboard, they take up no particular directions, but when a fairly large current is sent through the wire and the cardboard gently tapped, it will be seen that the filings form a series of concentric circles with the wire as centre, as in Fig. 106. The experiment can equally well be done with a number of small compass needles, which set themselves in the direction of the magnetic field which springs into being immediately the current is put on. It is found that the strength of the magnetic field in such a case, if the wire is a long one, varies inversely as the distance from the wire.

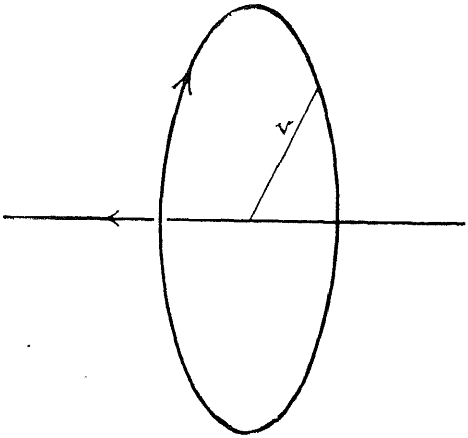

Fig. 107.

If the wire is bent round into a circle, the magnetic field at the centre of the circle due to the current is found to be at right angles to the plane of the coil, as in Fig. 107, every part of the current contributing to this effect.

If r is the radius of the circle, the strength of the magnetic field F is found to be directly proportional to the strength of the current C and inversely proportional to the radius.

If there are n windings, each of the same radius, then since each of the circular turns contributes to the magnetic effect as much as any other, the force becomes n times as big. If the factor of proportionality is taken as 2π, we may write—

$$F = \frac{2\pi nC}{r}.$$

Suppose, now, that a small magnet is suspended at the centre of a vertical coil so that it is free to move horizontally, it will of course set in the direction of the earth's horizontal field H; the vertical plane through the needle is the magnetic meridian and, if the plane of the coil is set so as to coincide with this plane, then when a current is sent through the coil, the magnetic field it produces acts at right angles to H. The result is that the needle is deflected through an angle in a manner very similar to what happens in the magnetometer experiment.

Here, again, if θ is the angle of deflection, we can at once say

$$\frac{F}{H} = \tan \theta.$$

Substituting for F the value above, this may be written

$$\frac{2\pi nC}{r} = H \tan \theta,$$

or,

$$C = \frac{Hr}{2\pi n} \tan \theta.$$

This gives a method by which the strength of electric currents can be measured, and it is of fundamental importance. Provided H is known, all that is required is accurate measurement of r and the tangent of the angle of deflection. An instrument based on these principles is called a tangent galvanometer. The latter word is derived from the name of a very early experimenter, Galvani, who long before the magnetic effects of currents were known, did remarkable work upon the physiological actions of electricity.

Since the magnetic force at the centre of a circular coil of wire carrying a current is so simply related to the current and to the dimensions of the coil, it provides a means of defining a unit current.

Suppose that a current C is flowing round n coils of radius r, the circumference of the wire is $2\pi r$ and each part of it contributes to the magnetic effect at the centre; the force F at the centre is given by $F=\frac{2\pi n C}{r}$. If unit current flows in the coil and the radius be 1 cm., then the force at the centre will be $F=\frac{2\pi \times 1}{1}=2\pi$ dynes. In order to get rid of the 2π and make the force at the centre the unit of force, *i.e.*, 1 dyne, all that need be done is to consider only that part of the current which flows round $\frac{1}{2\pi}$ part of the circumference, that is to say, along a length of arc equal to 1 cm., as in Fig. 108. The way in which the absolute C.G.S. unit of current is defined, therefore, is to say that it is that current which, flowing along 1 cm. of a circular coil of radius

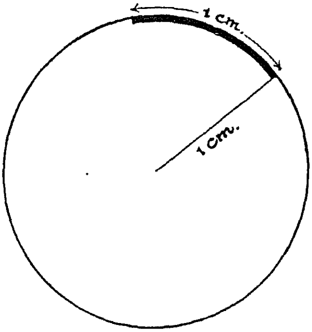

Fig. 108.

1 cm., produces at the centre a field of 1 gauss, *i.e.*, a force of 1 dyne per unit pole.

In practice, this is found to be rather a large unit, and the practical unit of current, called the ampere, is $\frac{1}{10}$ part of this unit, *i.e.*, ampere $= \frac{1}{10}$ C.G.S. unit of current.

Referring to the equation for the tangent galvanometer,

$$C = \frac{Hr}{2\pi n} \cdot \tan \theta,$$

it will be seen that $\frac{2\pi n}{r}$ depends on how the instrument is constructed, and is called the galvanometer constant G. Hence

$$C = \frac{H}{G} \tan \theta \text{ in C.G.S. units.}$$

$$= \frac{(10H)}{G} \tan \theta \text{ in amperes.}$$

$$= K \tan \theta \text{ in amperes.}$$

K is called the reduction factor of the galvanometer and is that quantity which, multiplied by the tangent of the angle of deflection, gives the current in amperes.

The tangent galvanometer is neither a very sensitive instrument nor a very convenient one to use, and many other types of instrument have been devised. If the magnet is suspended by a fine suspension and the angular deflection is measured by means of light reflected from a small mirror rigidly attached to the magnet suspension, it is much more sensitive; these improvements were made by Lord Kelvin in his mirror galvanometer. Since a coil of wire carrying a current behaves as a magnet, it can be used to replace the small magnet of the two previous instruments; the small coil when suspended between the poles of a large permanent horse-shoe magnet will, when a current flows through it, tend to rotate in accordance with the rules governing the action of one magnet on another; the amount of the deflection may in this way serve as a measure of the current passing through the suspended coil. Such an instrument is called a moving coil galvanometer (*vide* Fig. 109), and the strength of the magnetic field of the permanent magnet is so big that the action of the instrument is practically independent of the earth's field. All of the instruments mentioned require calibration before it can be stated what the current is, which gives a certain deflection of the moving part of the apparatus.

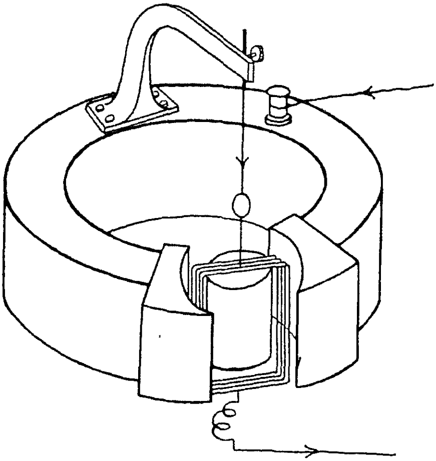

FIG. 109.

For many purposes it is very convenient to have direct reading instruments, that is to say, one in which the current in amperes, milliamperes, or microamperes can be read directly from a scale. The moving coil design is largely made use of in these instruments, and in such cases the moving coil is usually pivoted, not suspended, and its movement when a current flows is shown by a pointer attached to the coil, moving over a scale graduated to read the current directly.

As an ampere meter or ammeter is an instrument which is used in a circuit in order to measure the current flowing in it, it must have very little resistance, otherwise the value of the current would be changed on putting the instrument in the circuit.

OHM'S LAW.—When a current flows in a circuit, it behaves in many ways like an incompressible fluid. There must be something causing the current to flow; this is called the electro-motive force, or E.M.F. The bigger the E.M.F., other things remaining the same, the bigger the current flowing. In practically every circuit there is something opposing the passage of the current; this is called resistance; the bigger the resistance, the smaller the current flowing.

Ohm found by experiment that a simple relation holds between these three quantities, and this may be stated as follows: For a given conductor, the ratio of the potential difference between its ends to the current flowing through it is a constant for the conductor under given physical conditions.

If E is the potential difference and C the current, then $\frac{E}{C} = \text{constant} = R$, where R is the resistance of the conductor. Ohm's law applies not only to the whole circuit but to any part of it, so that if the potential difference E between any two points in a circuit is measured, the ratio of E to C will give the electrical resistance between the two points.

When water is pumped through the mains, work has to be done against the resistance to the flow of the water. In a similar way, when electricity is driven round a circuit, work is done against the resistance to the flow of the electricity, and this amount of work will of course vary with the electrical conditions of the circuit.

Unit potential difference (P.D.) is that potential difference which, in maintaining unit current in a circuit, performs a unit of work per second. This unit is much too small for practical purposes and, as already stated, the volt, a hundred million times as big, is the practical unit of potential difference or electro-motive force.

These two terms are not synonymous. In order to ·oduce an electrical current, some electro-motive force necessarily brought into the circuit, and this force ›es the work against the resistance in circuit, and this ill cause potential difference at different points of the rcuit. When a volt operating in a circuit causes a ırrent of an ampere to flow, the rate of expenditure of ıergy is said to be 1 watt; for big rates of working ıe term kilowatt is used.

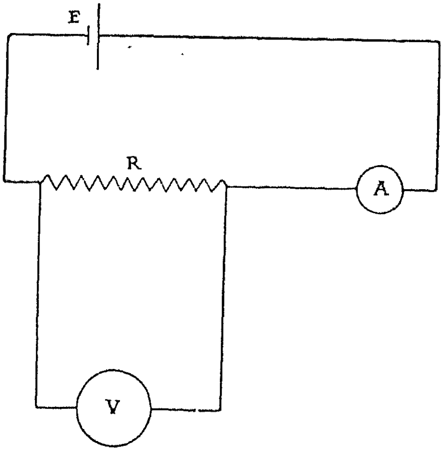

FIG. 110.

Instruments for measuring electro-motive force or otential differences are called voltmeters. Some ccount has already been given of these instruments uilt on electro-static lines, but the most usual type of oltmeter takes current and is of the moving coil onstruction. Suppose that a circuit consists of a ource of electro-motive force E which sends a current hrough a resistance R, the current being measured by n ampere-meter A, as in Fig. 110. If a voltmeter V is onnected to the two ends of the resistance, then, ırovided the resistance of V is large compared with R, relatively small amount will go through it and the instrument is calibrated to read the potential difference between the ends of R.

A current distributes itself round a circuit in such a way that there is nowhere any accumulation of electricity, and if alternative paths are open to the current, it chooses the path of least resistance.

Suppose that a current C flows in a circuit, and at the point A in Fig. 111 two paths of resistance R_1 and R_2 are open to it. The current divides, so that C_1 and C_2 are the two values of the current which at B becomes C again.

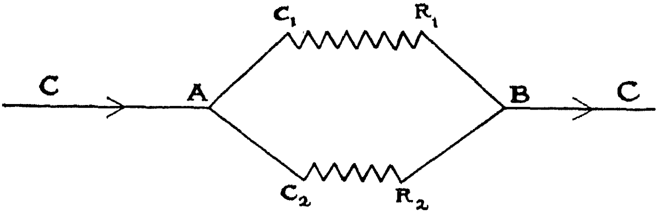

FIG. 111.

If E is the potential difference between A and B, then

$$C_1=\frac{E}{R_1}, \text{ and } C_2=\frac{E}{R_2}, \text{ and } C=C_1+C_2.$$

It is evident that whatever the resistance R_1 and R_2 may be, it would always be possible to substitute for them a single resistance which had the same effect; such a resistance is called the equivalent resistance R. If this resistance be supposed substituted for R_1 and R_2, then

$$C=\frac{E}{R}.$$

Hence,

$$\frac{E}{R}=\frac{E}{R_1}+\frac{E}{R_2}.$$

$$\therefore \frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}; \text{ or, } R=\frac{R_1R_2}{R_1+R_2}.$$

This method of using resistances is called putting them in parallel. If they are put one after the other, it is called putting them in series, and in this case the equivalent resistance is the sum of the individual resistances.

If resistances of 2 and 3 ohms are put in series, the equivalent resistance is 5 ohms; if they are put in parallel, it would be 1·2 ohms. A few examples will convince the reader that, when resistances are in parallel, the equivalent resistance is always less than the smallest of the individual resistances. If there are large numbers of resistances in parallel, the equivalent resistance is given by

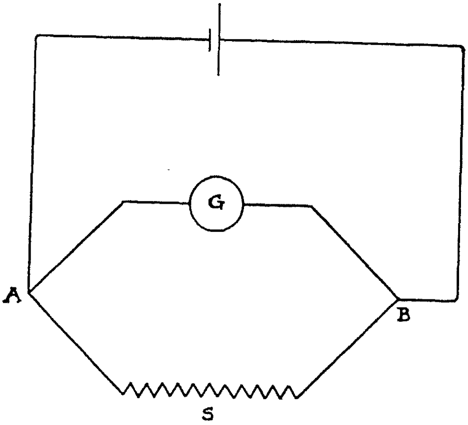

FIG. 112.

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}+\ \cdot\ \cdot\ \cdot$$

Suppose that a circuit consists of a galvanometer of resistance G and a resistance S in parallel which acts as a shunt, as in Fig. 112. Then since $\frac{GS}{G+S}$ is the equivalent resistance of the circuit, the current in the circuit will be

$$C = \frac{E}{\frac{GS}{G+S}},$$

where E is the potential difference between A and B. The current through the galvanometer will be

$$C_1 = \frac{E}{G} = C \cdot \frac{\frac{GS}{G+S}}{G} = C\left(\frac{S}{G+S}\right),$$

and similarly the current through the shunt will be

$$C_2 = C\left(\frac{G}{G+S}\right).$$

As an example, suppose that $S = \frac{1}{9}G$, then

$$G_1 = C\frac{\frac{1}{9}G}{G+\frac{1}{9}G} = \frac{1}{10C}.$$

In other words, if the shunt has one-ninth of the resistance of the current-measuring instrument, then only one-tenth of the current in the circuit passes through the instrument; the rest is shunted.

If $S = \frac{1}{99}G$, one-hundredth of the current goes through the instrument; and if $S = \frac{1}{999}G$, one-thousandth, and so on.

Measurement of Resistance.—The practical unit of resistance is called an ohm, and is defined as the resistance of a conductor carrying an ampere between the ends of which there is a potential difference of a volt. The standard ohm is the resistance at 0° C. of 14·452 grm. of mercury in the form of a column of uniform cross-section and 106·3 cm. long. Substandards of this have been made and copied, so that so-called standard ohms are available in most physical laboratories.

The most obvious way of measuring resistance is to pass a measured current through it and measure the potential difference operating in the circuit. The ratio of E to C gives R.

But in practice the measurement of resistance is nearly always made by comparing the unknown resistance with a known resistance. It is obvious that if a current which goes through an unknown resistance is measured when a definite voltage is applied to it, the value of the unknown resistance can be obtained; but in practice this is often a very inconvenient thing to do. The method which will be described here is due to Wheatstone and is known as the Wheatstone Bridge method. The principle of this method will be easily understood by an analogy.

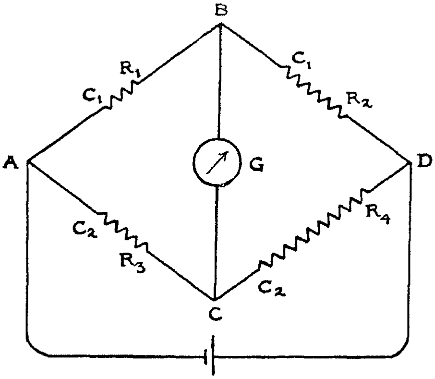

Fig. 113.

Suppose that in Fig. 113, ABD and ACD represent two different paths down a mountain, leading eventually to the common level D, then for any point, say B, in the path ABD, selected at random, it will always be found possible to find a point C in the path ACD which is at the same level as B. Now, if we have a circuit consisting of two alternative paths, ABD and ACD, for the current to flow along, and we select any point B on one of them, it will always be possible to find a point C on the other circuit at the same electrical level as B. When this point C has been found, an electrical connection made between B and C will not result in any current flowing along the circuit, because B and C are equipotential points. If, however, B is at a higher potential than C, electricity will flow from B to C, but if at a lower potential, it will flow from C to B. This, in fact, is how the point C is found; one uses a galvanometer to hit off the condition that no current flows either way.

Now let R_1, R_2, R_3, R_4 be the resistances of the four parts into which the circuit is divided when the condition that no current flows through the galvanometer is satisfied. Then since the potential drop from A to B is the same as that from A to C,

$$C_1R_1 = C_2R_3;$$

and similarly for the other part of the circuit,

$$C_1R_2 = C_2R_4.$$

$$\therefore \frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

This may be written

$$R_3 = R_4\left(\frac{R_1}{R_2}\right).$$

Hence the value of an unknown resistance R_3 can be found in terms of a known resistance R_4 and the ratio $\frac{R_1}{R_2}$ of two other resistances.

The most convenient form of the Wheatstone Bridge is that known as the post-office box.

The resistance of nearly all solid conductors increases with temperature. When a large current flows through an ordinary circuit, the circuit warms up considerably and overheating may occur. Carbon is an exception to the general rule, for its resistance decreases as it gets hotter.

If R_0 is the resistance of a conductor at temperature 0° C. and R_t the resistance at any other temperature t and α is the coefficient of increase of resistance, then the usual expression connecting the change of resistance with temperature is

$$R_t = R_0(1 + \alpha t).$$

This is found to hold approximately for most cases over moderate ranges of temperature.

WORK DONE BY ELECTRIC CURRENTS.—The work done in transporting a quantity of electricity Q in a circuit by means of an electro-motive force E is given by $W = EQ$.

If Q and E are measured in their appropriate practical units, namely, coulombs and volts, the result is so many joules, the practical unit of work. This can be deduced in the following way :—

1 coulomb $= 10^{-1}$ C.G.S. units of quantity.
1 volt $= 10^{8}$ C.G.S. units of electro-motive force.
$\therefore$ 1 volt coulomb $= 10^{-1} \times 10^{8}$ C.G.S. units of work.
$= 10^{7}$ ergs $= 1$ joule.

The usual evidence that work is being done when a current passes through a circuit is that heat is generated. In order to generate a unit of heat, *i.e.*, a calorie, by mechanical means, $4 \cdot 2 \times 10^{7}$ ergs or $4 \cdot 2$ joules must be expended, so that, as would be expected from this, 1 volt coulomb generates $\frac{1}{4 \cdot 2}$ calories, and experiments show that this is the case. This is sometimes referred to as the electrical equivalent of heat; it is, however, preferable to retain the term mechanical equivalent of heat, recognising at the same time that this may be achieved by electrical means; in this case the mechanism by which the work is done is not so obvious as in Joule's experiments.

It is in practice more convenient to express the coulombs transported in a circuit in terms of ampere

[redacted] [redacted]

seconds, and then the expression for the heat generated is

$$H = \frac{ECt}{4 \cdot 2} \text{ calories.}$$

Or, since

$$E = RC,$$

$$H = \frac{RC^2t}{4 \cdot 2} \text{ calories.}$$

Suppose that an electrical steriliser is connected to the 240-volt mains, that it takes 5 amperes and holds half a gallon of water at 15° C.; how long will it take to boil?

Heat will be generated at a rate given by

$$H = \frac{EC}{4 \cdot 2} \text{ calories per sec., } \textit{i.e.}, \text{ 286 calories per sec.}$$

Half a gallon is about 1·9 litres and will weigh 1·9 kg. The amount of heat required to raise 1·9 kg. of water from 15° to 100° C. is 160,500 calories. Hence the time required to generate this will be

$$\frac{160500}{286 \times 60} \text{ min., } \textit{i.e.}, \text{ about 10 min.}$$

This is evidently a minimum estimate of the time needed to heat the water, as some heat is taken up by the steriliser, some is lost by cooling to the surroundings, and, moreover, time is required for the conduction of the heat.

In many cases it is necessary to know not only the work done in a circuit, but the power or rate of expenditure of energy.

Since $W = ECt$ and power is the rate of doing work, it is evident that the power is given by $\frac{ECt}{t} = EC$. Hence, multiplying the volts by the amperes gives the power; this product is called a watt in honour of the engineer of that name. If 5 amperes are taken from a 200-volt supply, the power is 5×200 watts = 1 kw.

If this were continued for one hour, the amount of electrical energy consumed would be 1 kw.-hour.

In ordinary commercial and domestic use of electricity, the big electricity supply companies charge the consumer so much per kilowatt-hour ; this is called the Board of Trade unit.

ELECTRICAL THERMOMETRY.—Temperature can be measured electrically by two different methods. One method is to make use of the change in the resistance of a metal when its temperature changes ; the metal generally used for the purpose is platinum. The changes that occur in the resistance of this metal have been very carefully measured over a very much bigger range of temperature than is ever necessary in medical work. It has been found that the resistance R_t at any temperature compared with the resistance R_0 at a temperature 0° C. can be accurately found from the expression $R_t = R_0(1 + \alpha t + \beta t^2)$, where α and β are two constants. In contrary fashion, if the change in its resistance is found when the platinum is raised in temperature, then the change in temperature can be deduced. Platinum resistance thermometers, as they are called, are most often used for the measurement of relatively high temperatures, such as 1,000° C. or more, but they are sometimes used for the few degrees, say from 95° to 105° F., which are required in medical work. These thermometers are in fact of great use when a continuous record of the temperature changes is wanted, perhaps for a period of twenty-four hours ; and in this kind of work the platinum resistance is made the fourth arm of a Wheatstone Bridge and the deflections in the detecting galvanometer in circuit are made to give a continuous record on a revolving drum in a similar way to the record of a barograph.

The other method by which temperature changes can be measured electrically depends upon the phenomenon known as thermo-electricity. In 1826 Seebeck discovered that when two wires made of different metals are joined together, if there is a difference of temperature between the two junctions, a current is produced in the circuit and this current is maintained as long as the temperature difference is maintained. The electromotive force produced is called the thermo-electric E.M.F. Suppose that a piece of iron wire and a piece of copper wire are joined together and that one junction is put in a vessel containing melting ice, thus keeping its temperature constant; then if the other junction is at room temperature, a small E.M.F. is produced which can be shown by having a delicate milli-voltmeter in circuit. If the temperature of the warmer junction is now raised, the E.M.F. increases up to a certain point, but beyond this any further rise in temperature gives a smaller E.M.F. and eventually the E.M.F. is reduced to zero. If the junction be heated still more, the E.M.F. is actually reversed in direction. Heating the junction with an ordinary Bunsen burner is enough to cause this reversal of the thermo-electric E.M.F. in a copper-iron

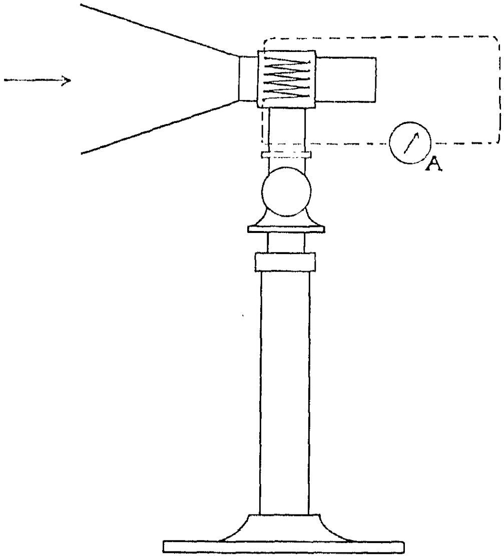

Fig. 114.

unction. Such a device is called a thermo-electric couple, and the E.M.F. given by a couple depends on the choice of metals as well as upon the difference of temperaure between the junctions. Antimony and bismuth are two metals which for a particular temperature difference give about the biggest E.M.F. If a number of couples are joined together in series, then they add heir E.M.F. in a similar way to a number of electrolytic cells. When about twenty antimony and bismuth couples are joined up in series, they make a very sensitive detector of radiant heat; the usual form that this takes s shown diagrammatically in Fig. 114. The junctions are packed together in a small compass, one set is overed over with a metal fitting which ends in a cone, hus leaving the other set of junctions exposed to any ource of radiant heat. The heat from a candle held a few yards away from the open end of such a thermocope can be easily detected. Professor Gamgee perected the thermo-electric method of recording body emperatures, but many rather elaborate precautions have to be taken when a continuous record of the body

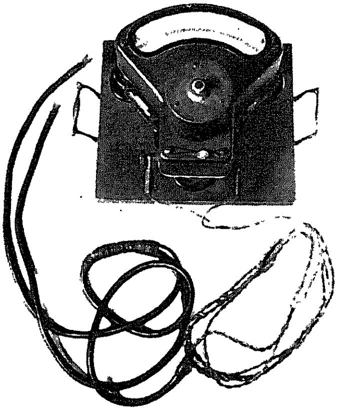

FIG. 115.

temperature accurate to a tenth of a degree is wanted. It is a much easier thing to measure differences of temperature between different parts of the body without necessarily knowing exactly what either temperature may be. A convenient type of instrument for this purpose is shown in Fig. 115. The thermo-couple is fixed in thin but strong strips of celluloid which allow them to be strapped firmly on to the parts of the body between which any temperature difference has to be measured.

MAGNETISM BY ELECTRICAL CURRENTS

Arago discovered that when a current was sent through a copper wire which was dipped in iron filings, they became attached to the wire. Each particle of iron becomes a magnet and sets in the direction of the magnetic field produced by the current; the north pole of the magnet is to the left of the current. This is a convenient rule to remember and was given by Ampère in his work on the magnetisation produced by currents. A bar of iron can be magnetised on these lines. The most convenient arrangement is to put the bar inside a tube on which a coil of wire is wound. When a current is sent through the wire, the lines of magnetism run in the direction of the length of the tube; they thread through the iron and thus magnetise it. The direction of the magnetisation is related to the direction of the current, as shown in Fig. 116. If the current is reversed, the poles of the magnet are reversed. The same rule applies in this case, viz., the north pole is to the left of the current. If an observer be supposed to be swimming in the direction of the current, then the north polar end of the magnet would be on his left.

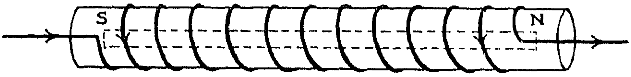

FIG. 116.

If a bar of steel is used for the above experiment, it will be found that it is still a magnet after the current is stopped. If soft iron is used, it is only a magnet while the current remains on; there is, however, often a small residual magnetisation left. Such a temporary magnet is called an electro-magnet; these are made in various forms and are of enormous importance in electro-technics.

Suppose that a long bar of steel is mounted inside a solenoid or helix, such as shown in Fig. 116, and that means are provided for sending as big a current as is necessary through the magnetising coils. Suppose, too, that the strength of the magnetisation of the iron rod is going to be tested by its action upon a distant magnetometer. It is found that beginning with a small current, the magnetisation of the iron rod gradually increases as the current is increased, as shown in Fig. 117; then proportionately more slowly until a condition of saturation of the iron is reached. At this point, any further increase in the magnetising current does not lead to any increase in the magnetisation of the iron. Suppose

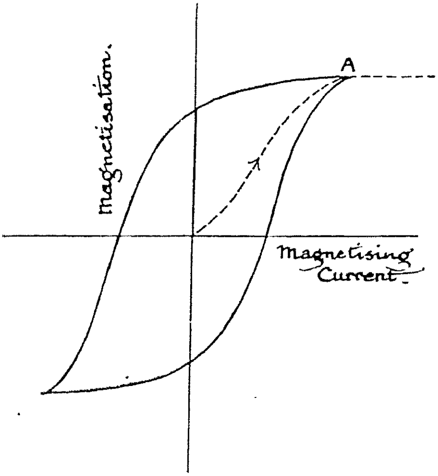

FIG. 117.

that the magnetising current is reduced when this point A on the curve is reached, then it is found that the magnetisation of the iron is not reduced to the initial corresponding value; in fact, when the magnetising current is zero, the iron will still be found to possess some magnetisation. In order to demagnetise the iron, the magnetising current will have to be reversed in direction and the general characteristics of the two curves are repeated once more, as will be seen from the diagram. It will be noticed that the magnetisation of the iron appears to lag behind the magnetisation current. Ewing, who discovered this phenomenon, gave it the name of hysteresis.

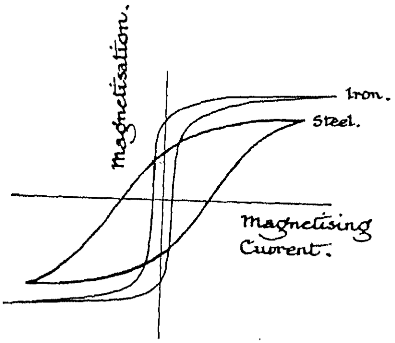

FIG. 118.

Fig. 118 gives the characteristic hysteresis loops for soft iron and for steel. It will be seen that when the magnetising current is suitably reduced, the permanent magnetisation of steel is much greater than that of iron. The fact that iron and steel can be magnetised to saturation point is of great importance in the design of electrical machinery.

FORCE OF A MAGNETIC FIELD ON A CURRENT.—An ordinary straight length of wire carrying a current is subject to a mechanical force if the wire is placed in a magnetic field, as shown in Fig. 119. This force acts at right angles both to the current and to the magnetic field, and so will tend to displace the wire at right angles to its length and to the magnetic field. If the direction of the current is reversed, then the displacement is reversed. A convenient rule by which to remember the direction in which the conductor would be displaced is to let the thumb, forefinger, and middle finger of the *left* hand point at right angles to each other; then, if the direction of the thumb is that of the current, the forefinger that of the magnetic field, then the middle finger gives the direction in which the conductor would tend to be displaced. This force is a very small one, and no movement will be detected if an ordinary copper wire carrying a few milliamperes is strung up in the magnetic field between the ends of two ordinary bar magnets. Einthoven has, however, succeeded in making what is called a string galvanometer by having the conductor of very thin silvered quartz and arranging that it is fixed in the field of a very strong electro-magnet. When a current passes through the quartz fibre, the fibre is displaced at right angles to its own length and to the magnetic field; the quartz fibre has so little inertia that it is practically " dead beat " * in action, and so responds at once to any variation in the current traversing it. In the instrument known as the electro-cardio-

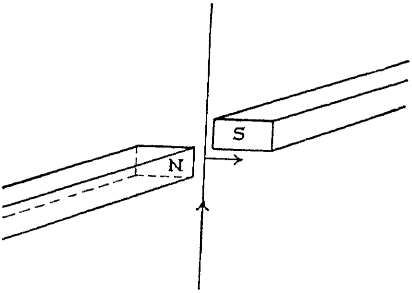

FIG. 119.

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* Dead beat means that when it is deflected, the fibre comes to its new position of equilibrium without oscillations or overswing.

graph, the displacements of the fibre are shown by means of an arc light and a projector which magnifies about 600 times. This makes it possible to record photographically the variations in what are called "heart currents," and a study of these variations has been of great service in diagnostic work on the heart.

ELECTROLYSIS

When electricity is passed through a liquid, it often happens that chemical changes take place. When this happens, the liquid is called an electrolyte and electricity is conveyed much more easily under these circumstances; if it does not occur, the liquid is called a non-electrolyte, and such liquids are all bad electrical conductors, in fact almost insulators. Mercury is an exception.

To Faraday we owe the first big advance in knowledge on this subject. He supposed that, when a substance like common salt, for instance, is dissolved in water, the act of solution causes large numbers of "ions" to be formed. A molecule of salt is written NaCl, and to indicate the change in the molecule which is brought about by dissolving the salt in water, it is written $\overset{+}{\text{Na}}/\overset{-}{\text{Cl}}$, the sodium atom assuming a positive and the chlorine atom a negative charge. The two atoms are still combined to form a molecule, but the effective linkage between them is so weakened that the atoms will be oppositely directed if a big enough electro-motive force is applied to them. Hence, if some salt solution is put into a vessel and two metal electrodes are dipped into the solution, when a current is passed through the solution, the sodium and chlorine ions travel through the liquid in different directions. Faraday termed this process the migration of ions, and it is this migration of the positive and negative components of the molecules of the electrolyte which accounts for the transit of electricity through it. The number of ions in an electrolyte is increased by an increase in the amount of solute in solution; but there is another process, namely, the ecombination of ions which always tends to limit he number of ions present. When the ions arrive at heir respective electrodes, the positive ions or anions oing to the negative electrode or kathode, and the egative ions or kations to the positive electrode or node, they lose their charges and are deposited on the lectrodes, sometimes appearing as gas and sometimes s solid; very often the deposited ions react chemically vith the solution. In the case of sodium chloride, he chlorine is to some extent liberated at the anode s gas; some goes into solution and the sodium reacts, orming NaOH.

When a current goes through a solution of copper ulphate, copper is deposited on the negative electrode s a thin film; by allowing the current to flow for long nough, this may be thick enough to cover the electrode nd the amount deposited may easily be weighed. Such process is called electroplating.

Faraday showed that the amount of a substance iberated in this way by electrolysis is proportional to he amount of electricity transported through the olution. Also by experiments on a large number of lectrolytes, he found a most important relation to old between the masses of substances liberated and heir chemical equivalents.

These conclusions may be summarised as follows :—

1. The mass of any substance liberated from an electrolyte by the passage of a current is directly proportional to the strength of the current and to the time for which it flows.
2. When the same current passes for the same time through different electrolytes, the masses of the different substances liberated are directly proportional to their chemical equivalents.

The first relation may be expressed as $Q = ZCt$, vhere Q is the amount of substance liberated by lectrolysis, C the current, and t the time for which t flows; Z is called the electro-chemical equivalent of the substance and is the amount in grams deposited by 1 amp. flowing for 1 sec., *i.e.*, by 1 coulomb.

The electro-chemical equivalents of a few substances are given in Table 8.

TABLE 8.

Substance.	Atomic Weight.	Valency.	Electro-chemical Equivalent.
Silver . . .	107·88	1	·0011183
Copper . . .	63·57	1 or 2	·0003295
Lead . . .	207·10	2	·001072
Hydrogen . .	1·008	1	·00001045

The copper in copper sulphate acts as a divalent atom; a single atom of it in this case carries twice as much positive charge as if it acted as a mono-valent atom, so that when transporting the same amount of electricity, only one-half as much of metallic copper will be deposited; so that, generally speaking, the amount of any substance deposited by a coulomb can only be calculated when the valency it assumes in the molecule is known.

When a current is sent through slightly acidulated water (say by weak sulphuric acid), bubbles of hydrogen and oxygen stream away from the kathode and anode which are preferably made of platinum. When collected by means such as shown in Fig. 120, the volume of hydrogen is found to be twice that of the oxygen in accordance with the chemical formula H_2O. When the products of electrolysis are gaseous, the presence of a layer of gas over the electrodes A and B greatly increases the resistance between them and also causes what is known as a back electro-motive force in the circuit. This phenomenon is sometimes called polarisation, and the E.M.F. caused by it, the E.M.F. of polarisation. This E.M.F. can be shown if a suitable switch is inserted in the circuit at CDE; when the current is put on, the current goes via EDB, causing evolution of gas at B and A. The switch at D is now put over in contact at C, a voltmeter at V being in this circuit. Immediately the switch is operated in this way, the voltmeter shows that an E.M.F. of about 1 volt occurs as a result of the platinum electrodes being covered with two different gases. If they were covered with the same gas, there would be no E.M.F.

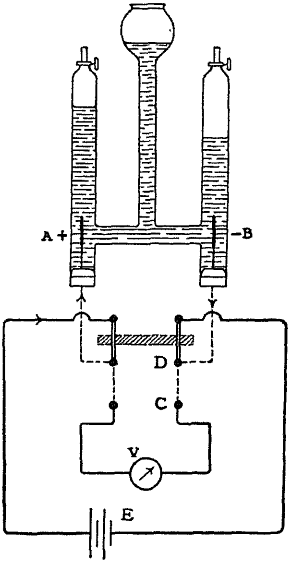

FIG. 120.

The result of this is that a current passes round the new circuit, but the back E.M.F. will quickly drop to a very low value.

The action resembles that of primary electrolytic cells. Two platinum electrodes dipped into sulphuric acid do not constitute such a cell, but if zinc were substituted for one of the platinum electrodes, then it would be found that an E.M.F. existed between the two electrodes, and this arrangement would be called a primary electrolytic cell.

A good many different varieties of these cells are known. Bunsen devised a cell in which carbon and zinc served as the electrodes with sulphuric acid as the electrolyte. The Daniell cell is an improvement on this; the electrodes are made of copper and zinc, the copper dipping into copper sulphate and the zinc into dilute sulphuric acid, the two liquids being separated by a porous pot; the E.M.F. of such a cell is about 1 volt and there is practically no polarisation. Another useful cell is the Leclanché cell; the electrodes are the same as in the Bunsen cell, being of carbon and zinc, but the electrolyte is ammonium chloride; a feature of the cell is that the carbon electrode is packed round with manganese dioxide, which helps the cell to recover from the polarisation which otherwise badly hinders its action.

All these primary electrolytic cells obtain electrical energy at the expense of the chemical energy of combination of the electrolyte used, so the cells need continual replenishment for this reason. Only very moderate currents can be got from these primary cells, as they all have an appreciable amount of resistance. A Leclanché usually has 1 or 2 ohms and a Daniell as much as 4 or 5 ohms.

Secondary cells are cells through which a current is first sent and, as a result of this, an E.M.F. is generated between the electrodes. When the primary current is stopped, this E.M.F. is then adequate to maintain a current. Gaseous hydrogen and oxygen covering two platinum electrodes is an example of a secondary cell, but it is not one for practical use. A vast amount of work has been done on such cells, and a highly efficient cell known as the lead accumulator is the outcome of it. To begin with, two electrodes of lead are put into fairly strong sulphuric acid (density 1·22) and a current is passed between them. Oxygen is liberated at the anode and this oxidises the lead plate to PbO_2; hydrogen bubbles away at the kathode. A potential difference of about 2 volts then exists between the two electrodes and, if the circuit is completed, a current will flow;

when this happens, the oxidised plate is reduced by the hydrogen to PbO, which with the acid in the cell tends to form $PbSO_4$. The current passes till the other electrode is oxidised and which also tends to form $PbSO_4$. This process is called forming the plates, the electrodes being in the form of plates, and when it is complete, the electrodes are both of lead sulphate in sulphuric acid, between which no E.M.F. exists.

On sending a current through the cell, the following changes occur :—

$$\left.\begin{array}{l} \text{At the positive plate,}\\ \quad PbSO_4+SO_4+2H_2O=PbO_2+2H_2SO_4\\ \text{At the negative plate,}\\ \qquad\qquad PbSO_4+H_2=Pb+H_2SO_4 \end{array}\right\rangle \text{Acid formed.}$$

On discharging the cell, the reactions are :—

$$\left.\begin{array}{l} \text{At the positive plate,}\\ PbO_2+H_2SO_4+H_2=PbSO_4+2H_2O\\ \text{At the negative plate,}\\ \qquad Pb+H_2SO_4+O=PbSO_4+H_2O \end{array}\right\rangle \text{Acidity diminished.}$$

It will be seen that when charging such a cell, acid is formed; in practice, the specific gravity may be allowed to rise as high as 1·22. On discharge, water is formed and the specific gravity may be allowed to fall as low as 1·18 before any damage to the cell is likely to happen.

These secondary cells or accumulators usually have very little internal resistance, so that appreciable current can be taken from them. The E.M.F. is just over 2 volts, and when discharging them it is inadvisable to let the voltage fall below 1·8.

THE MOTION OF IONS.—Ions travel comparatively slowly in electrolytes; we have to picture two opposing streams, directed by the electrical force applied between the electrodes and moving with a speed which depends upon the nature of the ion and upon the potential gradient in the liquid; the potential gradient is measured by the change in volts per centimetre through the liquid. The

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speed that an ion acquires under a potential gradient of 1 volt per cm. is called the mobility or migration velocity. Oliver Lodge made the first successful experiments in determining ionic mobilities. The arrangement he adopted was as shown in Fig. 121 :—

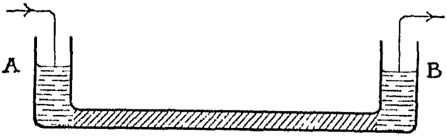

FIG. 121.

A narrow tube AB was filled with agar jelly mixed with sodium chloride and some phenolphthalein having a trace of caustic soda in it to colour it. The ends A and B were fitted into two small vessels holding sulphuric acid. When a current was passed in the direction AB, the hydrogen ions displaced the sodium ions, so that hydrochloric acid was formed; this acid decolourised the phenolphthalein, and, as more and more hydrogen ions were carried down the tube, this was shown by the gradual advance in the line of decolourisation. By measuring this rate, Lodge deduced that the mobility of the hydrogen ion was ·0026 cm. per sec.

Other methods have had to be used for measuring the mobilities of other ions, and the data in Table 9 show that the ions have widely differing mobilities.

TABLE 9.

Ion.	Mobility, *i.e.*, Speed in Centimetres per Second under a Potential Gradient of 1 volt per cm.
H	320×10^{-5} cm. per sec.
OH	182×10^{-5} ,,
CO_2	69×10^{-5} ,,
NH_4	66×10^{-5} ,,
K	66×10^{-5} ,,
Na	48×10^{-5} ,,

Considerable use is made in electrotherapy of many of the chemical ions; these ions can be passed through the skin and carried through the tissues by applying an electro-motive force between electrodes suitably placed.

Suppose that two pads are moistened with a solution of potassium iodide and strapped on to the back and front of the forearm. If an electro-motive force of 10 volts is applied between the electrodes, the potential gradient will be about 2 volts per cm., taking the thickness of the tissue to be 5 cm. Under these circumstances, the mobility of the iodine ion being ·067 mm. per sec., these ions will penetrate to a depth of about 4 mm. in ten minutes, provided they are not swept away by any of the fluids circulating in the vessels supplying the tissues.

As Lewis Jones has pointed out in his book, " Medical Electricity," it is extremely difficult to estimate completely the various influences of the movements of ions in the tissues, for it is certain that some degree of interchange occurs between the ions in the various tissues. Nerve tissue and muscle are rich in phosphates and potassium salts; the plasma and lymph have an appreciable content of chlorides and carbonates, so that in the passage of a direct current for an appreciable time through the tissues there will be numerous opportunities for some local alterations in the normal electrolytic content of the various structures.

The resistance of electrolytes, unlike most solids, diminishes as the temperature is increased.

CONDUCTION OF ELECTRICITY THROUGH GASES

Air and nearly all other gases at anything like atmospheric pressure conduct electricity very badly, but if the pressure is sufficiently reduced, their resistance to the passage of a current diminishes so much that they become fairly good conductors. This is of course a rather striking contrast to the conductivity of electrolytes, for as the amount of solute in a solution is increased, the current is more easily conducted.

For a long time it was thought that gases did not conduct electricity, but when means were devised for applying big electro-motive forces to them and it was found possible to lower their pressure at the same time, some very important information was obtained about the conditions under which gases could convey electric currents. As with liquids, the current is often conveyed by positive and negative ions; but sometimes the process appears to be quite independent of the gaseous molecules present, and then the current is carried entirely by particles of negative electricity called electrons which are much smaller than any atoms. The steps which led up to this discovery have been spread over a good many years and date back to 1857.

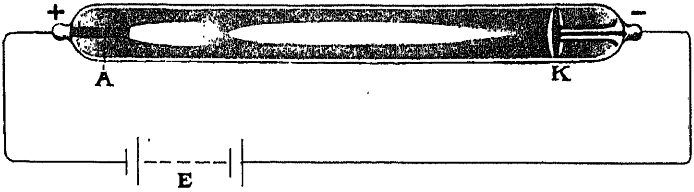

FIG. 122.

A convenient way of studying the conductivity of air or other gases, such as hydrogen or carbon dioxide, for instance, is to take a glass tube about 1 ft. long and 1 in. in diameter and seal two platinum electrodes in at the ends of the tube, as in Fig. 122, a small side tube of glass being sealed on so as to allow the apparatus to be connected to a suitable evacuation pump. If a small battery were put into such a circuit, then no appreciable current would pass through the tube, and even with a source of E.M.F. at E, giving several thousands of volts, it would still be difficult to show that any current was passing between A and B if the gas were at atmospheric pressure. Suppose, however, that 1,000 volts were applied between A and B and the exhaust pump were set working; a stage would soon be reached when a faint luminosity would be seen in the tube, and this would become stronger as exhaustion of the air went on.

As shown in Fig. 122, this spreads from very near the kathode K for almost the whole length of the tube and is called the negative column; a non-luminous space separates this from another glow which reaches up to the positive electrode and is called the positive column.

If the pressure is reduced to just less than 1 mm. of mercury, the appearance of the discharge changes to that seen in Fig. 123; the luminous positive column appears to be split up into segments which are called striæ, which look like a number of discs set side by side along the length of the tube.

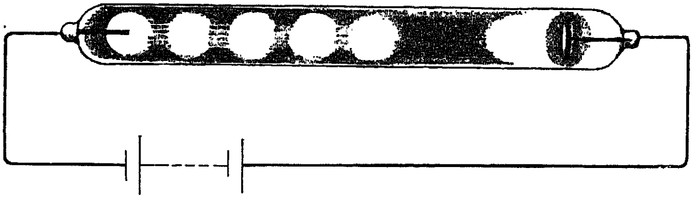

FIG. 123.

The dark space which immediately surrounds the negative electrode is called the Crookes dark space; there is another dark space called the Faraday dark space which is near to, but separated from, the Crookes dark space by a luminous region as shown in the figure. When the pressure in any of these discharge tubes is reduced low enough, a new phenomenon is seen, and this consists of a faint luminosity which spreads away from the kathode; this is called the kathode stream, and for a long time the nature of this stream remained unknown in spite of a great deal of experimental work. Crookes himself thought that the kathode stream had nothing to do with the atoms or molecules of the gas left in the discharge tube; in fact he went further than this and expressed the view that it was a "fourth state of matter." Long before this prediction about the nature of the kathode stream, Plucker had shown that the stream carried a negative charge of electricity, and some years later Perrin and Schuster showed that the direction of the stream could be changed by the action of electrical and magnetic fields, and that the stream always moved in the same way as a stream of negatively electrified particles would move. From these numerous experiments, round about 1895 the view generally held was that the kathode stream consisted of negatively electrified particles, but that these particles were not atoms or molecules of the gas in the discharge tube. If not atoms or molecules, what were they? At that date nothing smaller than an atom or molecule was known to science. This matter is dealt with further in Chapter IX.

CHAPTER VIII

ELECTRO-MAGNETISM

ELECTRO-MAGNETIC INDUCTION.—Due no doubt to his discovery of electro-static induction, Faraday was convinced that some similar phenomenon existed between electrical currents. He devised many experiments in order to find out whether this was so, and he succeeded in showing that when a current was started or stopped or changed in any way in one circuit, then it could induce a current in a neighbouring circuit. It is usual to call the first the primary circuit and the other the secondary circuit.

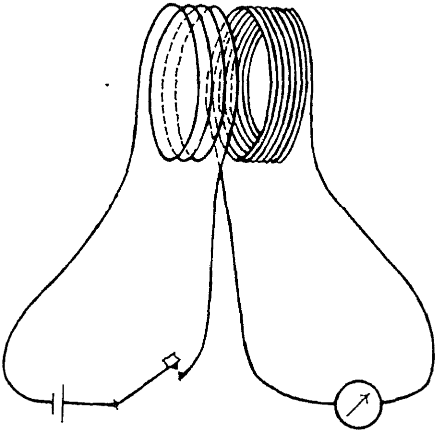

FIG. 124.

The phenomenon can be well shown by an arrangement similar to that in Fig. 124. The primary circuit consists of a few coils of thick wire through which a few amperes can be sent from a storage cell, a key being inserted in circuit for starting or stopping the current.

The secondary circuit consists of a completely distinct circuit consisting of many turns of thin wire and placed near to, but not touching, the primary windings; the rest of this circuit includes a detecting galvanometer.

Though the two circuits are not connected electrically, it is found that when a current in the primary is started, a momentary current flows in the secondary; when the current continues to flow steadily in the primary, no effect whatever is seen in the secondary. If, however, the current in the primary circuit is broken, a momentary current flows in the secondary but in the opposite direction to that first seen in that circuit.

Faraday showed that the phenomenon was due entirely to the magnetic field of the primary circuit. An essential part of the experimental arrangement is that this magnetic field shall in some way thread through the secondary, and in this way the whole effect becomes fairly easily explicable. Suppose that an E.M.F. is applied to the secondary circuit, causing a current to flow in it, the immediate effect of this is to make a magnetic field spring into existence, linked with the circuit in a way which has been mentioned in Chapter VII. Hence, if a magnetic field is sent through the secondary circuit, it may be expected that this will *per contra* set up an E.M.F. in the circuit, and that is exactly what happens.

Faraday showed that for any particular setting of primary and secondary circuits—and there are, of course, innumerable ways of arranging them—the induced E.M.F. in the secondary depends simply upon the rate at which the magnetic lines through the secondary are changed.

If N_1 represents the " flux of induction " threading through the secondary circuit at a time t, and if N_2 represents the flux of induction threading through the secondary circuit at a time t_2, due to any changes produced, then the rate of change of the lines is given by $\frac{N_1 - N_2}{t_1 - t_2}$ and the induced E.M.F. is proportional to this quantity.

The flux of induction itself is proportional to the area through which any magnetic field passes and is measured by HA, where H is the strength of the magnetic field and A the area. In this case the flux is AH units. If H were 1 gauss and A 1 sq. cm., the flux of induction would be unity.

Suppose in an actual case the area were 100 sq. cm. and the value of H 1,100 gauss, and that one second later the value of the magnetic field was reduced to 100 gauss, then the rate of change of the flux of induction comes to be $\frac{100\ (1100 - 100)}{1} = 100{,}000$ absolute units.

Since the electro-motive force induced in the circuit is proportional to this rate of change, the actual E.M.F.* in such a case would be one millivolt. It will readily be understood, therefore, that theoretically there is no limit to the E.M.F. which can be produced by means of electro-magnetic induction.

When it comes to the practical application of this principle, certain difficulties crop up. There is no particular difficulty in the production of strong magnetic fields, but to change their strength rapidly cannot be done without the fullest consideration being given to all the electrical and magnetic parts of the circuit. This is well exemplified in the action of an induction coil. In its simplest form, this consists of a helix having two or three windings of thick copper wire; the interior of the helix is filled with iron wire or strips lying along the length of the helix. When a current is sent through the windings, a magnetic field is produced inside the helix and this is enhanced by the iron core. Another helix having a large number of windings envelops the primary but is not electrically connected with it; this is the secondary circuit. The action of the instrument may be briefly described as follows: a current is sent through the primary circuit and this induces an E.M.F. in the secondary circuit. This E.M.F. is only momentary, for directly the current in the primary is fully established, the electro-magnetic induction stops. It is, however, arranged that the current in the primary is made and broken very frequently by some form of interrupter; in this way a succession of alternate increases and decreases in the strength of the magnetic field threading through the numerous windings of the secondary is made and maintained. For every interruption there is an E.M.F. generated in the secondary. For several reasons the E.M.F. generated when the current in the primary is broken is greater than when it is made. In coils used for generating X-rays, it is purposely arranged that the E.M.F. at "break" is many times as big as it is at "make"; in other coils, such as, for instance, the Helmholtz coil used in physiological work, it is arranged that the E.M.F. generated at "make" is just equal to that at "break." There are various kinds of interrupters; one of the earliest forms is called the hammer break, the action of which may be seen from Fig. 125, which indicates the essential features of an X-ray induction coil. A is a platinum

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* One volt is equal to one hundred million absolute units of electro-motive force, *vide* Appendix, p. 226.

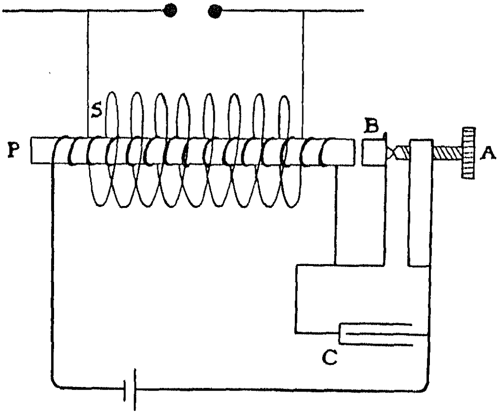

FIG. 125.

contact which just touches B, which is a piece of soft iron mounted on a piece of steel spring. When the current passes through the primary, the iron core becomes magnetised and this attracts the piece of soft iron strongly enough to drag it away from A; the current is thus broken. The iron core is no longer magnetised, B is no longer attracted to it, and the steel spring brings it back into contact with A and the current is made again. A condenser C is put in the circuit as shown, for it is found that, if a suitable capacity is used here, it enhances the E.M.F. at "break" and diminishes it at "make," a very important requirement in X-ray work. The exact way in which the capacity acts in such a circuit is a complicated one, but it may suffice to say here that, roughly, the action of the condenser may be thought of as follows. When the primary circuit is made, the condenser is short circuited. With the breaking of the primary current the back induced E.M.F. in the primary coil tends to prolong the current, which if the condenser is there, will flow into it, charging it to a fairly high potential difference in opposition to that of the source supplying the coil. Lord Rayleigh concluded that the only value of the condenser was to take up this extra current at break, the back E.M.F. of the condenser thus making the interruption more sudden than would be the case in the absence of the condenser. Provided the break could be made sudden enough by other means, the condenser would then become unnecessary. These conclusions were supported by his well-known experiment in which the interruption was made by a bullet fired from a rifle, which produced a distinctly higher secondary potential than could be obtained from an ordinary interrupter and condenser. The theory which takes his name assumed that when current flowed in the secondary of an induction coil, it did not interact on the primary circuit. Taylor Jones showed that when allowance is made for the back action of the secondary circuit on the primary coil, there was an optimum value for the primary condenser, and a repetition of the rifle bullet experiment gave, with his apparatus, a much smaller secondary potential than that given by an ordinary interrupter fitted with a condenser of optimum capacity. Crowther has shown that the induction coils in frequent medical use conform to one or other of these two modes of action.

The ratio of the E.M.F. in the two circuits of an induction coil can be found as follows: If E_1 is the E.M.F. in the primary and C_1 the current, then E_1C_1 is the energy or rate at which work is done in that circuit. The work that could be got out of the secondary is at most what is put into the primary, so, if E_2 and C_2 are the E.M.F. and current in the secondary, it may be said that E_2C_2 cannot be greater than E_1C_1, therefore $\frac{E_2}{E_1}$ cannot be greater than the ratio of $\frac{C_2}{C_1}$. Suppose, for instance, that the voltage on the primary coil is 100 volts and that 8 amp. pass through it, and that the current in the secondary is 5 milliamp., then the maximum E.M.F., when this happens, will be

$$\frac{100 \times 8}{5 \times 10^{-3}} = 160000 \text{ volts.}$$

This is not an uncommon voltage to use in X-ray work.

In practice, the hammer break is rather noisy; it is not very rapid in action, and when the current is broken there is often considerable arcing at the platinum contacts. Other interrupters have been devised, one of the commonest being the mercury interrupter. In this device a conical spindle drilled with two holes running from the inverted apex to the periphery is spun rapidly in a pool of mercury by means of a small motor; this causes some mercury to pass up the spindle, and it then splashes out in two opposite jets at right angles to the axis of the spindle; metal contacts are placed so that these two jets fall on them, making a conducting path from one contact to the other. The current is thus made through the primary, but as the spindle is rotated, the mercury jets now miss the fixed contacts and the current is broken. These mercury interrupters can work at high speeds of interruption and so help to generate a big E.M.F. in the secondary circuit.

There are many experimental ways of showing the production of an induced E.M.F. If the effect is due to the magnetic field threading through a secondary circuit, then the primary circuit of Fig. 124 can be replaced by a magnet. This can be shown by taking a few coils of wire connected up to a sensitive galvanometer and suddenly putting one end of a strong bar magnet into the coils; an E.M.F. is produced giving a transient current. When the magnet is withdrawn, a current in the opposite direction is produced.

An important relation exists between the direction of the current circulating in the secondary and that in the primary. This is conveniently expressed by Lenz's law:—

"The direction of an induced current is always such that by its electro-magnetic action it tends to oppose the displacement."

The phenomenon of electro-magnetic induction is the basis upon which the production of electrical currents on a big scale depends. The usual form of mechanical contrivance which produces electrical currents is called a dynamo, and the simplest form of this instrument is called an earth inductor, the action of which is as follows:—

A wooden framework (*vide* Fig. 126) about 1 ft. in diameter is wound with about 300 turns of wire and is mounted so that it can turn about a vertical axis; the two ends of the windings are brought out along this axis and are connected to a detecting galvanometer. If the earth inductor is set up in such a way that the magnetic field of the earth sweeps through it to a maximum extent, then the effect of rotating the inductor is to alter this linkage of the magnetic field with it, and an E.M.F. will be induced in the circuit, causing a current to flow through it. When the framework has been rotated through 90°, its plane will coincide with the direction of the horizontal component of the earth's field and so no magnetic lines thread through it. On rotating still further, lines once again begin to thread through until the maximum number goes through; further rotation again brings a condition of no flux through the inductor and the final 90° to complete one revolution sees the number of lines of magnetic force linked with the circuit once again back to its maximum value.

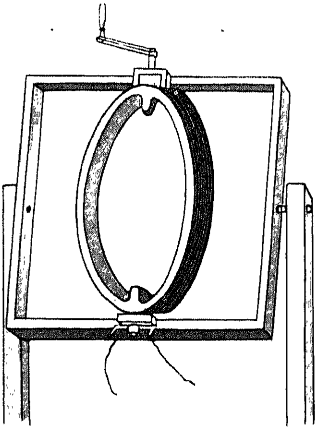

FIG. 126.

In this way one complete revolution of the inductor leads to the generation of an E.M.F. which varies in magnitude and direction in the way shown in Fig. 127. Such a current is called an alternating current, and the rate of its alternations will depend on the rate of revolution of the inductor. On the basis of this action, machines for the generation of electrical currents have been constructed which are in fact used all over the world for lighting and power purposes. The construction of such machines designed for giving a large output requires a very great deal of consideration of an electro-technical character. Strong electro-magnets are used instead of the weak magnetic field of the earth, and a number of circuits just out of step with each other are used instead of a single circuit, so that directly the E.M.F. in one is declining in value, it is supplemented by the next circuit following it in rotation. In this way the current though alternating does not show the violent fluctuations indicated in Fig. 127.

The converse action would be to have a circuit or series of circuits capable of revolving in an extraneous magnetic field, but instead of giving it a mechanical motion and so generating a current, to send a current through the circuits and thereby cause their rotation in the magnetic field. This is the action of an electric motor.

Telephone.—This instrument, devised by Graham Bell, depends on electro-magnetic induction for its action. A long horse-shoe shaped electro-magnet made of soft iron, round which are wound a number of turns of wire for conveying the current, is fixed in a tube, at one end of which a very thin steel plate is mounted which acts as a diaphragm. This diaphragm is put very near to one end of the electro-magnet and forms part of the magnetic circuit, so that some of the lines of magnetic force thread through it. Any variation in position of this diaphragm produces an alteration in the magnetic flux threading through the coils of the electro-magnet and will therefore produce an E.M.F. in the circuit. This variation in position of the diaphragm is produced by the compression and rarefaction of the air when one talks in front of the diaphragm—the circuit of Fig. 128 is completed by a similar instrument which acts as receiver. This induced and varying E.M.F. causes a varying current in the second part of

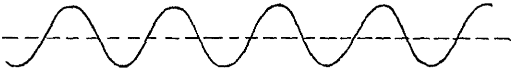

Fig. 127.

the circuit, reproducing on its own diaphragm exactly the vibrations which the speaker's voice gives rise to. In this way speech is reproduced and can be transmitted over long distances.

A large number of different types of currents are used for medical purposes, and these give rise to very different effects. For instance, a direct current passed from hand to hand can only be tolerated if the current is kept small, that is to say, a few milliamperes; this method of giving a current is called galvanism, or the use of a galvanic current. If there is an interrupter in the circuit, so that the current is made and broken several times a second, it is called an interrupted current. If such a current is sent through the primary coil of an induction coil and the current taken from the secondary terminals, this is a current of high voltage in which the impulses only last a very small interval of time, and so momentary currents pass through the body, stop, then pass again, and so on; these currents are called faradic currents. Again, an ordinary alternating current which reverses its direction about fifty times a second cannot be tolerated any more than a direct current; this is often called a sinusoidal current. If, however, by special means the alternations are made so rapid that they occur with a frequency of about 1,000,000 a second, much larger currents can be tolerated by the body; these currents are called diathermy currents and

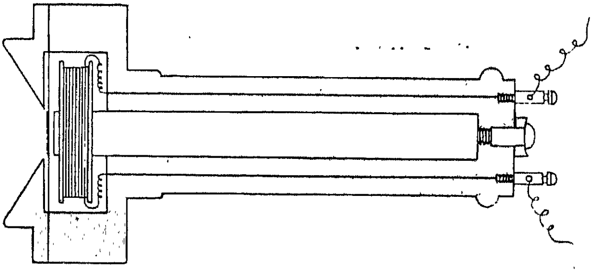

FIG. 128.

are used to produce heat in any part of the body. If the terminals of a diathermy machine are held in the hands, a current of as much as an ampere can be tolerated, and after a little while a distinct sensation of heat is felt at the wrists and up the arms. Such currents are often used in medicine, and it is possible to localise this heating process so that the tissues are actually coagulated. An excellent account of the use of these currents is given by E. P. Cumberbatch in his book "Medical Diathermy."

CHAPTER IX

X-RAYS AND RADIO-ACTIVITY

X-RAYS.—In 1895 Röntgen discovered a new kind of radiation to which he gave the name X-rays. He discovered that these rays were emitted from a Crookes tube, and found that the rays had the remarkable property of being able to penetrate many things which are opaque to light. Ordinary black paper, for instance, offers practically no hindrance to the passage of X-rays, and the tissues can be penetrated to such an extent, that by suitable means one can see into or through many parts of the body. Röntgen found that the X-rays were somehow or other produced by the kathode stream and, although the exact way in which this occurs is still not known, the fact remains that when the kathode stream hits an obstacle placed in its path, X-rays originate at the place where the stream is stopped. This led to an important alteration in the design of an X-ray tube, so that the kathode stream was focused on to the anti-kathode; this was done by having a concave kathode and putting the centre of the anti-kathode near its centre of curvature. A tube made like this is called a focus tube, one of which is shown in Fig. 129.

The real nature of the particles in the kathode stream was discovered by J. J. Thomson in 1897, two years after the discovery of X-rays. He found that they were not ordinary atoms at all but simply minute particles of negative electricity; he suggested the name corpuscle for these particles, but a few years later the name electron was given to them. The electrical charge that each electron carries is often called the elementary charge; it is a negative charge and just equal in value though opposite in sign to the charge carried by the hydrogen ion in electrolysis. The hydrogen atom was, until 1897, the smallest known mass, but the electron is far smaller than this; measurement of its mass shows that it has only about one sixteen-hundredth of the mass of the hydrogen atom.

Since the discovery of electrons, several methods of producing them have been found. The filament of an incandescent lamp when lit is found to emit these electrons, and the valves so largely used in wireless work depend for their action upon the stream of electrons being easily directed by an electrical field in one direction but not in the opposite direction, so that the current is uni-directional. However electrons are generated they are found to be of exactly the same kind; practically all metals when heated give out electrons from their surfaces, and this suggests that the electron is really a constituent part of these substances. The opinion generally held about them is that they are a most important constituent of all matter, and since the electron is a negative unit of electricity, it has to be explained how it is that matter still so often remains electrically neutral.

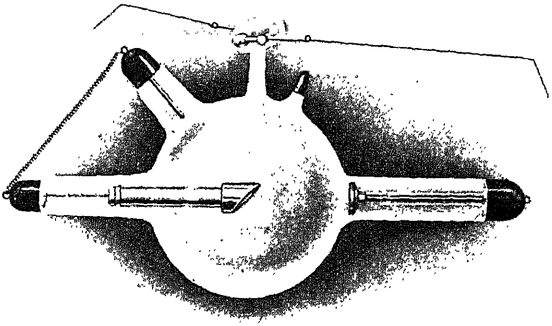

FIG. 129.

The main facts of electrolysis prove that in liquid conduction, positively charged ions play just as important a part as negatively charged ones, and yet no experiment has succeeded in showing that minute particles of positive electricity, the exact counterpart of the negative electrons, exist. Yet the charge on the hydrogen ion and on a good many other varieties of positive ions is only just equal to that of the electron though opposite in sign; so it seems that the unit of positive charge is always linked with some mass of atomic size and does not exist free like electrons. This has led to the view that the main mass of an atom is concentrated where the positive charge is.

Though discovered in 1895, the true nature of X-rays was not proved until 1913, when it was shown that they resemble light, being a sort of wave motion in the æther but of very much shorter wave-length. Between these two dates a great deal was discovered about the physical properties of X-rays and about the best ways of producing them. It was found, for instance, that if an E.M.F. of about 50,000 volts are applied to an X-ray tube, the X-rays generated have only moderate penetrating power through the tissues of the body; this could be shown by the fact that with such X-rays a very good radiograph is got of the bones of the hand, but a poor one of the hip-joint where a much greater thickness of tissue has to be penetrated.

Further experiments showed that, if 100,000 volts were applied to the same tube, the X-rays generated were much more penetrating, and this sort of relation is known to hold over a big range of voltage—the higher the voltage, the more penetrating the X-rays produced; after 1913 it was possible to add the words "the shorter the wave-length."

X-rays are generated by applying a high E.M.F. to an X-ray tube. This is supplied by various forms of electrical machinery, such as an induction coil or transformer; but under no circumstances does it appear possible to apply a high E.M.F. to an X-ray tube and produce X-rays of a single wave-length. What always happens is that heterogeneous X-rays covering a range of wave-lengths are produced, just as when a match is struck the light produced covers a considerable range of the visible spectrum. The range of wave-lengths actually produced depends upon the voltage applied. Hence the important thing to know when applying a definite E.M.F. to an X-ray tube is what range of wave-lengths is produced and what is the wave-length of the shortest.

The following relation has been found between the maximum voltage V applied to an X-ray tube and the minimum wave-length λ (in Ångstrom units (*vide* p. 115)) produced :—

$$V=\frac{12400}{\lambda}; \text{ or, } \lambda=\frac{12400}{V}.$$

Example.—If the voltage applied to the terminals of an X-ray tube were 124,000, then the shortest X-rays produced would have a wave-length of ·1 Å.U.

The air in an ordinary X-ray focus tube is generally pumped out so that the pressure is not more than about ·01 mm. of mercury, and the air that is left in the tube plays an important part in facilitating the production of the kathode rays which are then directed, by the E.M.F. applied to the tube, on to the anti-kathode. If the pressure of the air is reduced still further so that the pressure is less than ·001 mm. of mercury, the kathode stream is not properly formed and no X-rays are produced.

Sometimes the kathode stream is produced in an entirely different way. It has been mentioned that by heating a metal wire to a high temperature, electrons are given out by the wire. This is made use of in the form of X-ray tube called the Coolidge tube, where the kathode consists of a flat spiral of tungsten wire. Current from a small battery of lead accumulators is sent through the spiral, raising it to a white heat, and the electrons showered out of it are then directed on to the anti-kathode by the E.M.F. applied between the two terminals of the tube.

There is no essential difference between the X-rays produced by these two different methods, provided the high potential E.M.F. is the same.

Air is a bad electrical conductor, but when a beam of X-rays is passed through it, the conductivity is enormously increased; this process is called ionisation and it consists in the creation of positive and negative gaseous ions. This occurs in air, or in fact in any other gas through which the rays go.

So striking is this effect that it is not a difficult matter with delicate instruments to measure the ionisation produced 100 yards away from an X-ray tube. This process of separation of the two kinds of electricity doubtless occurs in solids and liquids too, and very probably plays an important part in the biological changes which can be produced by X-rays.

The two other chief physical characteristics of X-rays are their photographic and fluorescent action. X-rays have a very similar photographic action to light, so that X-ray photographs, or "radiographs" as they are called, can be produced by interposing the object between the X-ray tube and the plate and recording the shadows produced; in order to get good definition, the object is put as close to the plate as possible. After the exposure, the plate is treated on the same general lines as in ordinary photography. Fig. 130 is a reproduction of a radiograph of a hand, and the details of its structure are well shown.

Fluorescence is the emission by a body of rays of a different character to those incident upon it. For instance, if a colourless solution of quinine sulphate is exposed to a beam of ultra-violet rays, the solution gives out light of a lavender-blue colour; by some means it absorbs some of the ultra-violet rays and transforms them into fluorescent visible rays. Gabriel Stokes found that a general rule was followed in the vast majority of such cases, namely, that the wavelength of the fluorescent rays is always longer than the incident rays. This is true also for X-rays; these rays are not visible to the naked eye, but a good many substances are made visible by putting them in the path of a beam of X-rays. A cardboard screen covered with a layer of barium platino-cyanide crystals lights up very brightly with an apple-green colour under these circumstances, and these screens are largely used for viewing purposes in the medical uses of X-rays.

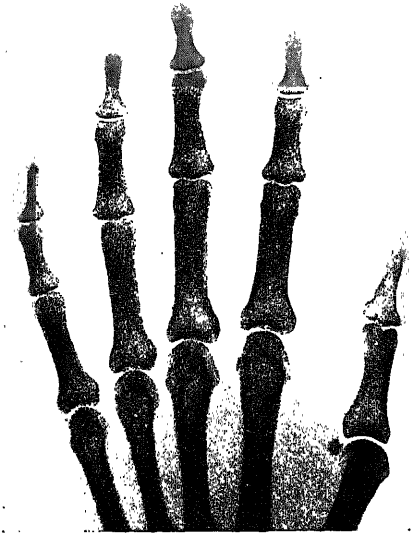

FIG. 130.

RADIO-ACTIVITY.—Not long after X-rays were discovered, Henri Becquerel found that some of the compounds of uranium appeared to give out rays which had properties rather like X-rays. These rays had the power of ionising the air through which they went, and they could penetrate things ordinarily opaque to light—they would, for instance, easily give a photographic impression upon plates which were wrapped in black paper. Becquerel undertook his investigations with the definite intention of seeing whether any substances under natural conditions gave out rays like X-rays spontaneously, and he discovered that this really did happen. At first he thought that it was something to do with the particular compounds of uranium which he started with, all of which were fluorescent; but by carefully planned experiments, he showed that the fact that it was a compound had nothing to do with the phenomenon, for the rays were given out by the metal uranium itself. Becquerel gave the name radio-activity to this phenomenon, the spontaneous emission of rays by a body. Soon after this discovery in 1897, Professor and Madame Curie found two other radio-active substances to which they gave the names polonium and radium. Since then a great deal of work has been done on the subject, with the result that nearly forty radio-active bodies are now known.

The rays given out by radio-active bodies are of three distinct kinds called alpha, beta, and gamma rays. Alpha rays are not really rays in the sense in which the word ray is generally used, for they are positively charged helium atoms which are shot out of the atoms of radio-active substances. These particles generally travel with about a tenth of the velocity of light, but they do not go more than a few inches, as the numerous collisions they make with the molecules of the air very quickly stop them; an ordinary sheet of notepaper is enough to stop them. In spite of the short distance they travel, they ionise the air through which they go very strongly; they can affect a photographic plate and cause many substances to fluoresce very brightly.

Beta rays are electrons, exactly the same as those in the kathode stream but generally travelling with higher speeds; some beta rays travel with very nearly the speed of light. They are not so easily stopped as alpha rays and can penetrate air, water, paper, and thin sheets of metal like aluminium, copper, and silver. Even the swiftest beta rays are, however, completely stopped by 2 mm. of lead or 1 mm. of platinum. These rays also have ionising, photographic, and fluorescent action.

Gamma rays are very similar to X-rays, the only difference being that they generally have considerably shorter wave-length than X-rays. They are, however, not only of one wave-length but range over a considerable region of the spectrum, being comprised between about ·4 Å.U. and ·01 Å.U. The shortest X-rays which have yet been produced have a wave-length of about ·05 Å.U.

The production of alpha, beta, and gamma rays by radio-active bodies is an entirely natural process. The beta and gamma rays from a milligram of radium sealed up in a thin glass tube can be fairly accurately measured, and if ten times the amount of radium be taken, the yield of rays is found to be exactly ten times as great.

The fact that alpha rays are helium atoms suggests either that helium is always mixed up with the radio-active substances that emit alpha rays, or that we are here witnessing a very remarkable change in the atoms of which these substances are made; for if a helium atom does really come out of a radio-active atom, this must lead to a different atom being formed. This is what is believed to happen, for radio-active bodies actually change one into the other; for instance, solid radium changes very gradually into a different substance called radium emanation or radon, which is a gas, and this in turn gradually changes into another substance called radium A, which is a solid, and so on; the changes do not stop here but go on, as may be seen from the series of substances which are arranged in column 1 of Table 10.

TABLE 10

THE RADIUM SERIES

Substance.	Atomic Weight.	Time to Decay to Half Value.	Rays Emitted.
Radium . .	226	1,680 years	Alpha and beta.
Radium emanation, or radon	222	3·86 days	Alpha.
Radium A . .	218	3 minutes	Alpha.
Radium B . .	214	26·5 minutes	Beta and gamma.
Radium C . .	214	19·5 minutes	Alpha, beta, and gamma.
Radium C′ and C″ .	214	10^{-6} seconds 1·9 minutes	Alpha. Beta.
Radium D . .	210	16·7 years	Beta.
Radium E . .	210	5 days	Beta and gamma.
Radium F (polonium)	210	136 days	Alpha.
End product . .	206	Non-radio active	..

In other columns will be found some information about the rays which these different bodies emit and the relative times which they take over the transmutation from one substance to another. In the case of radium, the change of this substance into radon is so slow that it can hardly be measured in a direct way, and the time mentioned in Table 10 is an indirect estimate rather than a measurement. In the case of radon, this substance changes into radium A more rapidly, and the rate at which it changes can be accurately measured. Suppose, for instance, that the activity of some of this radio-active gas at any moment is 100 units, then in almost exactly 3·86 days it will be found to have diminished to a strength of 50 units; in another 3·86 days this activity will be reduced again to one-half, namely, 25 units, and so on, always taking 3·86 days to diminish to one-half of its strength. This time is called the time period of the radio-active body; it is characteristic of the particular body concerned and there is no known way in which it can be altered. It will be seen that if a body follows this simple law in the way in which it gradually decays, theoretically it never entirely vanishes, because however often we may halve a positive quantity, a small positive quantity will always remain. When the time period is a matter of minutes, as it is for instance in the case of radium A, then after, say, a day, the amount of it remaining unchanged is too small to be detected.

The alchemists of the last century tried to transmute the baser elements into the more precious; during this time Nature was doing not quite this, but something analogous, in many minerals which were scattered about the earth's crust. The importance of radio-activity in many natural phenomena is gradually being recognised, and the use of beta and gamma rays in medicine in the treatment of disease is becoming a very important branch of radio-therapy.

The discovery of X-rays and radio-activity and the isolation of the electron, all within a few years of each other, mark an epoch in physics comparable with those created by the researches of Isaac Newton, Michael Faraday, or Clerk Maxwell.

APPENDIX

ELECTRICAL UNITS

It has been explained in previous pages that three sets of units are used in electrical work. It is, of course, important to know what relation these bear to one another. The electro-static units have been derived from a knowledge of the phenomena associated with the existence of static charges; the electro-magnetic units have been derived from a knowledge of what happens when electricity is put into motion; both of these sets of units are largely used in many branches of scientific work, but at the same time the need was felt for more practical units, and these are either multiples or submultiples of the units derived from fundamental considerations.

In Table 11 these three different sets of units will be found grouped together, and the value of any one unit in any of the three systems may be found by inspection.

Table 11.

Quantity.	Name of Practical Unit.	Value in C.G.S. Units.	
		Electro-static.	Electro-magnetic.
Quantity of electricity	Coulomb	10^{-1} V	10^{-1}
Strength of current	Ampere	10^{-1} V	10^{-1}
Electro-motive force	Volt	$10^{8} \div V$	10^{8}
Resistance	Ohm	$10^{9} \div V^{2}$	10^{9}
Capacity	Farad	$10^{-9}V^{2}$	10^{-9}

$V = 3 \times 10^{10}$ cm. per sec.

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